I-B-ECHO: Diffracted echoes and finite supplied-report instruments

Front coefficients, windowed deficit separation, holonomy channels, and finite-precision core indistinguishability on the prescribed Enter spacetime

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Abstract

We study the apex-diffracted part of the massless Friedrichs Green operator on the prescribed conical product used in the Enter spacetime framework. The cone, its deficit, the bundle holonomy, and the extension are inputs. The continuum-to-register preparation is a separate assumption; a destructive finite instrument does not acquire an unknown continuum amplitude.

Starting from the explicitly stated I-B-G kernel, we derive the general twisted front coefficient and its high-frequency asymptotic. For the untwisted scalar, the leading coefficient agrees, in the declared Fourier convention, with the difference-angle term of Keller's wedge coefficient. The remainder is stated additively, including when the leading coefficient vanishes.

Two admissible time windows define a direct-pulse integral and an echo integral. Their calibrated ratio is a nonnegative weighted average of a closed scalar function. On $1/2 < \alpha \leq 1$, it vanishes exactly at zero deficit. On every compact parameter sector avoiding backscatter shadow values, including sectors with $\alpha < 1/2$, appropriately isolated windows satisfy explicit separation estimates. A sufficiently short, nonzero window also gives an injective averaged functional on each such sector. Supplied finite preparations, the X-J signal instrument, and complete X-K composition then give a total-variation lower bound with explicit implementation-error allowances. The

actual classical outputs are finite codes and stochastic symbols, not exact real amplitudes.

For neutral smooth curvature cores, we prove agreement before a round-trip core time and convergence of spatially smeared records. We give a quantitative record-error bound in terms of exterior resolvent discrepancies and derive the associated Bessel matching coefficient. An explicit profile-uniform decay rate still requires additional estimates. The resulting non-reconstruction conclusion is finite-precision indistinguishability, not a theorem that finitely many exact real functionals are necessarily non-injective.

Twisted-scalar and spin-1 front identities are retained with their channel conventions. Twisted-core, vector-core, and curvature-regular spin-2 echo extensions remain open at specified interfaces. Numerical discrepancies from the supplied version are retained as unexecuted reports, not certificates. Standing refusals are frozen in the I-B-G Book Ledger v1; O.1 remains NOT_CLOSED; no downstream quantum admission or sourcing claim is made.

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Book edition note (t427u)

Standing Enter refusals, scope vocabulary (proved here/supplied input/conditional/ not constructed/not claimed), the four-layer instrument box, and Scholar bibliography policy are frozen in the companion chapter *I-B-G Book*

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Book expansion: separation table, theorem promotions, four-layer cite

toprule Landmark Relation Separation

midrule I-B / En09-A Destructive clock / drop identity Residual retention does not change the old channel

I-B-G SCD / front / supported transfer Continuum kernels ≠ field-state certificates

I-B-G-S Spin-1 front / image identities Curvature-regular spin-2 front still OPEN

VP-5 lineage Neutral smooth-core pre-arrival agreement Comparison only; not a premise of TV bounds

Keller (1962) Difference-angle wedge term Cone has no reflecting-face term; twist not in Keller

X-J / X-K Signal channel / composition Finite instruments; no continuum acquisition

bottomrule

Four-layer instrument box

Cite the I-B-G Book Ledger four-layer box (continuum host / window functional / supplied preparation / finite instrument). This sheet occupies Layers 2–4 once Layer 1 kernels are supplied by I-B-G.

Theorem promotions (book edition)

The twisted front coefficient (formerly Lemma lem:front) and the windowed deficit-separation estimates (Theorem thm:readout and Theorem thm:finiteinjectivity) are treated as numbered theorems of the book corpus. The complete-record total-variation lower bound under X-J / X-K composition is Theorem thm:TV:

$$\mathrm{TV}\left(\widetilde{P}_{\alpha_1}, \widetilde{P}_{\alpha_2}\right) \geq \left[\frac{(1-r)|\nu|}{2\sigma}\operatorname{sech}^2\left(\frac{|\nu|B_J}{\sigma}\right)\cdot\operatorname{pos}\left(\frac{\pi}{2}|\alpha_1 - \alpha_2| - 2E_J\right) - 2e\right]_+.$$

Marginalization cannot increase TV, so the full recorded history inherits at least this gap.

Front coefficient versus Keller; VP-5 smooth-core pointer

Untwisted scalar D_0 matches Keller's difference-angle contribution; the cone omits the reflecting-face term. Twisted D_β is the holonomy counterpart (not claimed inside Keller). Neutral smooth-core pre-arrival agreement with the VP-5 lineage remains a *comparison*: it does not certify curvature-regular spin-2 fronts and does not discharge residual-retention obligations.

Scope, evidence, and the release contract

The standing operational boundary

This companion separates four objects:

boxed
prescribed continuum host and Green operator,
real window functionals of a retarded field,
supplied finite preparations and report words,
complete finite instruments and their record laws.

An implication between these objects is used only when its hypotheses are stated. In particular, a finite instrument is not a continuum-to-register detector.

The finite operational interpretation follows the Enter calculus and the prescribed-host companions of Cheung [1, 2, 9, 7, 8]. The relevant clock and signal channels are reproduced and proved below. Their use does not remove external preparation, calibration, or scheduling time. A classical record does not determine the surviving quantum residual.

The non-sourcing boundary is equally important. The metric is not solved for, varied by the instrument, or obtained from an Einstein equation. A conserved continuum tensor, if separately available, is not thereby a source of this cone [2, 10].

Evidence labels

Definition (Evidence and construction status). The following labels apply throughout.

\textsf{ANALYTIC} A mathematical statement proved here, or an explicitly identified inherited analytic input. For inherited inputs, the label does not assert an independent audit of the entire source.

\textsf{SUPPLIED} A version-qualified source description, preparation specification, bibliographic record, or computational report supplied with the manuscript.

\textsf{UNCERTIFIED} An approximation or computational claim lacking the artifacts and total error enclosure needed for certification in this release.

\textsf{OPEN} A construction or estimate not supplied here. This means ``not established in this release,'' not ``proved impossible.'' Definitions are conventions rather than empirical claims. Every numbered mathematical result below is marked ANALYTIC; assumptions and documentary reports are marked separately.

A supplied numerical report may have SUPPLIED provenance while its claimed accuracy remains UNCERTIFIED. These labels are not collapsed. This follows the residual-retention discipline of XVI-O1-ST [11].

Dependencies and non-transfer

The scalar analytic input is I-B-G [3]. The spin-channel interpretation is taken only at the level explicitly stated from I-B-G-S [4]. Neutral curvature-core selection is related to I-B-VP-5 [5]; a proof for the restricted class used here is included. The exterior matching calculation below is derived directly and does not rely on an unavailable detailed theorem from I-B-VP-5-Ex1 [6].

The finite channels are those of I-B, En09-A, X-J-ST, and X-K-ST [2, 9, 7, 8]. Their finite-dimensional algebra does not import the analytic content of an unrelated continuum construction.

Oai282-A is cited only for the finite-test coverage distinction [12]. No underlying OpenAI catalogue theorem is a premise. The operator-versus-contact distinction and the characteristic quantum-admission problems described in the supplied XIV/XV records are not altered by this free scalar calculation [13].

SUPPLIED programme boundaries. This manuscript makes no status update to another manuscript or registry. In the supplied programme terminology, O.1 remains NOT_CLOSED; XVI-N is not discharged; the downstream quantum-admission exit called XV-B is not opened; Problems 1, 2, and 8 remain unfinished. The manuscript-local H2, I3, and J2 obligations receive no new discharge here. This wording preserves the distinction between XIV-H's historical H2 stop and XIV-I's separately reported I2 internal-amplitude construction. A technical manuscript called XV-B may exist while the distinct downstream admission exit remains unopened [13, 14, 15].

The prescribed host and the scalar kernel

Assumption (SUPPLIED: host, bundle, and extension). Let

$$0 < \alpha \le 1, \qquad \mu = 2\pi(1-\alpha), \qquad \beta \in [0,1),$$

and set $c = 1$. The regular spacetime is

$$M = \mathbb{R}_t \times (0,\infty)_r \times (\mathbb{R}/2\pi\mathbb{Z})_\varphi \times \mathbb{R}_z, \qquad g = -\mathrm{d}t^2 + \mathrm{d}r^2 + \alpha^2 r^2 \mathrm{d}\varphi^2 + \mathrm{d}z^2.$$

A twisted scalar is represented on the angular cover by

$$v(r,\varphi+2\pi,z) = e^{2\pi i\beta} v(r,\varphi,z).$$

The spatial operator is the nonnegative Friedrichs extension $A_{\mathrm{F},\beta}$ of the covariant negative Laplacian. The retarded wave operator is defined spectrally by

$$G_+(t) = \theta(t)\frac{\sin\left(t\sqrt{A_{\mathrm{F},\beta}}\right)}{\sqrt{A_{\mathrm{F},\beta}}},$$

with the multiplier assigned its continuous value t at spectral value zero. The metric, bundle, and extension are prescribed inputs [2, 3].

All spatial kernels below are relative to $\mathrm{d}V_0 = \alpha r\,\mathrm{d}r\,\mathrm{d}\varphi\,\mathrm{d}z$. The time variable in a kernel denotes elapsed time.

For a lifted angular difference $\theta = \alpha(\varphi - \varphi')$, put

$$K(\theta) = \{k \in \mathbb{Z} : |\theta - 2\pi\alpha k| < \pi\},$$

$$R_k^2 = r^2 + r'^2 - 2rr'\cos(\theta - 2\pi\alpha k) + \Delta z^2,$$

$$\rho(u)^2 = r^2 + r'^2 + 2rr'\cosh u + \Delta z^2, \qquad \rho_0 = \rho(0).$$

Input statement (ANALYTIC input: scalar kernel). For a regular spatial pair off the shadow set

$$\theta \pm \pi \in 2\pi\alpha\mathbb{Z},$$

the I-B-G massless kernel is

$$G_+(t; x, x') = \sum_{k \in K(\theta)} e^{2\pi i\beta k} \frac{\delta(t - R_k)}{4\pi R_k} - \theta(t - \rho_0) \frac{\Phi_\beta(u_0, \theta)}{4\pi^2 \alpha r r' \sinh u_0},$$

where

$$\cosh u_0 = \frac{t^2 - r^2 - r'^2 - \Delta z^2}{2rr'}.$$

For $u > 0$,

$$\Phi_\beta(u, \theta) = \sum_{n \in \mathbb{Z}} \sin\left(\frac{\pi|n + \beta|}{\alpha}\right) e^{-|n + \beta \mathrm{rvertu}/\alpha} e^{\mathrm{i}(n + \beta)\theta/\alpha}.$$

With

$$a = \frac{\pi}{\alpha}, \qquad b = \frac{u}{\alpha}, \qquad \xi = \frac{\theta}{\alpha},$$

define

$$F(\gamma, w) = \frac{1}{2\mathrm{i}}\left(\frac{e^{\gamma(w + \mathrm{i}a)}}{1 - e^{w + \mathrm{i}a}} - \frac{e^{\gamma(w - \mathrm{i}a)}}{1 - e^{w - \mathrm{i}a}}\right).$$

Then

$$\Phi_\beta(u, \theta) = F(\beta, -b + \mathrm{i}\xi) + F(1 - \beta, -b - \mathrm{i}\xi).$$

In particular,

$$\Phi_0(u, \theta) = \frac{1}{2}\sum_{\pm} \frac{\sin((\pi \pm \theta)/\alpha)}{\cosh(u/\alpha) - \cos((\pi \pm \theta)/\alpha)}.$$

The diffraction weight vanishes identically if and only if

$$\alpha^{-1} = N \in \mathbb{N}, \qquad N\beta \in \mathbb{Z}.$$

The Friedrichs wave group has intrinsic finite propagation.

At shadow lines, the geometric and diffracted terms must be interpreted through their total joint distribution, not by separately substituting into singular front formulas. These are the scalar inputs used from I-B-G [3, eq:Phi-closed, eq:Carslaw, thm:closed, thm:speed].

The conic diffraction framework is classical [17, 18]. The contribution here is the explicit functional-to-instrument interface and its scoped error statements, not a claim that apex diffraction itself is new.

Finite pulses and arrival decomposition

Fix emitter and receiver positions

$$x_e = (r_e, \varphi_e, 0), \qquad x_r = (r_r, \varphi_r, 0), \qquad r_e, r_r > 0.$$

Let $f \in C_c^\infty(\mathbb{R})$ satisfy

$$f \geq 0, \qquad \operatorname{supp} f \subset [0, T_s], \qquad \int_{\mathbb{R}} f(t)\,\mathrm{d}t = 1.$$

The point-source response below is an evaluation of the explicitly given kernel after temporal convolution. The later core theorem uses spatially smeared sources and receivers instead.

Proposition (ANALYTIC: pulse decomposition). *At a non-shadow spatial pair,*

$$u(t) = \sum_{k \in K(\theta)} e^{2\pi i \beta k} \frac{f(t - R_k)}{4\pi R_k} + u_{\mathrm{dif}}(t),$$

$$u_{\mathrm{dif}}(t) = -\frac{1}{4\pi^2 \alpha} \int_0^\infty \Phi_\beta(u, \theta) \frac{f(t - \rho(u))}{\rho(u)}\,\mathrm{d}u.$$

For $\Delta z = 0$, the diffracted term vanishes for $t < L := r_e + r_r$. This possible onset time does not depend on α or β, although the term can vanish identically. Every geometric length in $K(\theta)$ is strictly less than L.

If

$$T_s < L - \max_{k \in K(\theta)} R_k,$$

all geometric pulses end before the diffracted contribution can begin.

Proof. Convolve with f. In its diffracted part set $t - s = \rho(u)$. Since

$$\rho'(u) = \frac{r_e r_r \sinh u}{\rho(u)},$$

the Jacobian cancels $r_e r_r \sinh u$, giving . The support of f and $\rho(u) \geq L$ give the onset statement. Finally,

$$R_k^2 = L^2 - 2r_e r_r(1 + \cos(\theta - 2\pi\alpha k)) < L^2$$

because the defining angular inequality is strict.

Remark (ANALYTIC: the second ray). For $\alpha > 1/2$, in the principal lift $0 \leq \theta \leq \pi\alpha$ and off shadow lines, $k = 0$ is present. A second ray, $k = 1$, is present exactly when $\theta > \pi(2\alpha - 1)$. The equality of the two lengths at $\theta = \pi\alpha$ is not the condition for the second ray to appear.

Front coefficients and the high-frequency limit

Theorem (ANALYTIC: evenness and twisted front value). *Away from its meromorphic singularities,*

$$F(\gamma, w) = F(1 - \gamma, -w).$$

Consequently, at a non-shadow angle, $\Phi_\beta(u, \theta)$ extends as an even analytic function of u near zero and

$$\Phi_\beta(0, \theta) = \frac{1}{2} e^{\mathrm{i}(\beta - \frac{1}{2})\xi} \left[\frac{e^{\mathrm{i}(\beta - \frac{1}{2})a}}{\sin((a + \xi)/2)} + \frac{e^{-\mathrm{i}(\beta - \frac{1}{2})a}}{\sin((a - \xi)/2)} \right].$$

In particular,

$$\Phi_0(0, \theta) = \frac{\sin a}{\cos \xi - \cos a} = \frac{1}{2} \left[\cot \frac{a + \xi}{2} + \cot \frac{a - \xi}{2} \right],$$

$$c_0(\alpha, \beta) := \Phi_\beta(0, 0) = \frac{\cos((\beta - \frac{1}{2})\pi/\alpha)}{\sin(\pi/(2\alpha))},$$

$$\Phi_{1-\beta}(0,\theta) = \overline{\Phi_{\beta}(0,\theta)}.$$

The possible denominator singularities are on the shadow set ; some are removable. No assertion is made that every shadow value is a genuine pole.

Proof. Rewrite each denominator in $F(1-\gamma,-w)$ using

$$1 - e^{-w \pm \mathrm{i}a} = -e^{-w \pm \mathrm{i}a}(1 - e^{w \mp \mathrm{i}a}).$$

The two terms exchange, proving the first identity. Applying it to the two terms in proves evenness.

At $u = 0$, use

$$\frac{1}{1-e^{\mathrm{i}x}} = \frac{\mathrm{i}e^{-\mathrm{i}x/2}}{2\sin(x/2)}.$$

Substitution into , followed by addition of its two contributions, gives . The denominators are nonzero off , proving analyticity there. Setting $\beta = 0$, or $\xi = 0$, gives and . Replacing β by $1-\beta$ conjugates the displayed phases, which gives .

For untwisted backscatter,

$$c_0(\alpha, 0) = \cot\frac{\pi}{2\alpha}.$$

At $\alpha = 1$,

$$c_0(1,\beta) = \sin(\pi\beta).$$

Theorem (ANALYTIC: additive high-frequency asymptotic). *Fix $r_e, r_r > 0$, $\Delta z = 0$, and a non-shadow angle. Use the temporal Fourier convention*

$$\widehat{v}(k) = \int_{\mathbb{R}} e^{\mathrm{i}kt} v(t)\,\mathrm{d}t, \qquad k > 0.$$

For the impulsive diffracted kernel,

$$\widehat{G}_{\mathrm{dif}}(k; x_r, x_e) = \frac{e^{\mathrm{i}kL}}{4\pi\sqrt{r_e r_r L}} D_{\beta}(\theta; k) + O(k^{-3/2}),$$

where

$$D_\beta(\theta;k) = -\frac{e^{\mathrm{i}\pi/4}}{\alpha\sqrt{2\pi k}}\,\Phi_\beta(0,\theta).$$

The error is uniform on fixed compact sets of regular non-shadow parameters. If $\Phi_\beta(0,\theta) \neq 0$*, the leading term may equivalently be multiplied by* $1 + O(k^{-1})$*.*

For $\beta = 0$*,*

$$D_0(\theta;k) = \frac{e^{\mathrm{i}\pi/4}\sin(\pi/\alpha)}{\alpha\sqrt{2\pi k}(\cos(\pi/\alpha) - \cos(\theta/\alpha))}.$$

This is the difference-angle contribution in the usual scalar wedge coefficient for opening angle $\pi\alpha$*, with the present time convention. The cone has no reflecting face term [16]. For* $\beta \neq 0$*, is its holonomy-twisted counterpart, not a claim that Keller's untwisted wedge problem already contains the additional bundle parameter.*

Proof. Write $t = L + s$, $s \geq 0$. The identity

$$(2r_e r_r \sinh u_0)^2 = (t^2 - L^2)(t^2 - (r_r - r_e)^2)$$

gives

$$r_e r_r \sinh u_0 = \sqrt{s}\, h_1(s), \qquad h_1(0) = \sqrt{2Lr_e r_r},$$

with h_1 analytic near zero. By Lemma , $\Phi_\beta(u_0,\theta)$ is analytic in s near zero. Thus

$$G_{\mathrm{dif}}(L+s) = s^{-1/2}h(s), \qquad h(0) = -\frac{\Phi_\beta(0,\theta)}{4\pi^2\alpha\sqrt{2Lr_e r_r}}.$$

Choose a smooth cutoff equal to one near $s = 0$. The endpoint expansion of its Fourier integral begins with

$$h(0)\int_0^\infty e^{\mathrm{i}ks}s^{-1/2}\,\mathrm{d}s = h(0)\sqrt{\pi}\,e^{\mathrm{i}\pi/4}k^{-1/2}.$$

Subtracting this term leaves $O(k^{-3/2})$, by the next endpoint term. The cutoff complement is handled by repeated integration by parts. The closed weight decays exponentially in u, while $\rho(u)$ grows exponentially, so the required time derivatives are integrable at infinity. This proves the additive estimate and its compact-set version. These are

the standard endpoint Fourier estimates [25].

Regrouping the leading constant gives –. Division by a nonzero leading coefficient gives the relative statement; such division is not made when the coefficient vanishes. Finally, gives .

Remark (ANALYTIC: normalization of the wedge comparison). A scalar wedge with Dirichlet or Neumann faces is obtained from the doubled cone by antisymmetric or symmetric reflection. Its coefficient therefore contains both difference-angle and sum-angle terms. The cone coefficient is the difference-angle term, not the full coefficient of either boundary-value problem. Equality in is equality of the leading coefficient; it is not an exact frequency-domain solution at finite k.

Remark (ANALYTIC: small-deficit comparison). Let $\Delta = \pi(1-\alpha)$ and $\theta = \pi\alpha - \theta$. Then

$$-\frac{\Phi_0(0,\theta)}{\alpha} = -\frac{1}{2\alpha}\left[\cot\frac{\Delta-\theta}{2\alpha} + \cot\frac{\Delta+\theta}{2\alpha}\right].$$

The plane-wave cosmic-string treatment of Fernández-Núñez and Bulashenko [20] uses the corresponding two half-plane diffraction contributions. Termwise expansion away from a shadow line gives the same leading inverse-angle behavior. A relative comparison is not uniform near cancellation of the two terms. Exact wave propagation on cosmic-string backgrounds is also studied in [19]; no numerical identification with that paper's normalizations is claimed here.

Admissible windows, including image sectors

From now through Section , use untwisted backscatter:

$$\beta = 0, \qquad \varphi_r = \varphi_e, \qquad 0 < r_e < r_r, \qquad R_0 = r_r - r_e, \qquad L = r_e + r_r.$$

The excluded backscatter shadow values are

$$\mathscr{S} = \left\{ \frac{1}{2m} : m \in \mathbb{N} \right\}.$$

The connected sectors of $(0,1] \setminus \mathscr{S}$ are

$$\mathscr{I}_0 = (1/2, 1], \qquad \mathscr{I}_m = \left(\frac{1}{2m+2}, \frac{1}{2m}\right), \quad m \geq 1.$$

Lemma (ANALYTIC: image count and isolation margins). *For $\alpha \in \mathscr{I}_m$,*

$$K(0) = \{-m, \ldots, m\}.$$

There are exactly $m + 1$ distinct geometric lengths,

$$R_j(\alpha) = \sqrt{r_e^2 + r_r^2 - 2 r_e r_r \cos(2\pi\alpha j)}, \qquad j = 0, \ldots, m,$$

and

$$R_0 < R_1 < \cdots < R_m < L$$

when $m \geq 1$.

Let J be a compact interval contained in $\mathscr{I}_m$. Define

$$g_{\text{tail}}(J) = \min_{\alpha \in J} (L - R_m(\alpha)) > 0,$$

where $R_m = R_0$ for $m = 0$, and

$$g_{\text{in}}(J) = \begin{cases} +\infty, & m = 0, \\ \min_{\alpha \in J} (R_1(\alpha) - R_0) > 0, & m \geq 1. \end{cases}$$

If $T_s < \min(g_{\text{in}}, g_{\text{tail}})$, one common direct window isolates the $k = 0$ pulse for every $\alpha \in J$, and every geometric pulse ends before L.

Proof. The condition on k is $|k| < 1/(2\alpha)$. For $\alpha \in \mathscr{I}_m$, the allowed nonnegative integers are $0, \ldots, m$. The angle $2\pi\alpha j$ lies in $[0, \pi)$ on that range, so is strictly increasing in j. The margins are positive by continuity and compactness. The pulse-duration inequalities then give disjoint support margins.

Definition (Continuum window functionals). Fix one compact sector interval $J \subset \mathscr{I}_m$ and a pulse obeying Lemma . Choose bounded nonnegative windows χ_d, χ_e such that:

1. $\chi_d = 1$ on $[R_0, R_0 + T_s]$, and its support lies strictly before both L and every non-direct geometric arrival for $\alpha \in J$;

2. $\operatorname{supp}\chi_e \subset [L, L+T_w]$, where $T_w > 0$. Define the continuum values

$$y_d = \int \chi_d(t)u(t)\,\mathrm{d}t, \qquad y_e = \int \chi_e(t)u(t)\,\mathrm{d}t.$$

Put

$$W(u) = \int_{\mathbb{R}} \chi_e(t) f(t-\rho(u))\,\mathrm{d}t,$$

$$\cosh u_* = \frac{(L+T_w)^2 - r_e^2 - r_r^2}{2r_e r_r},$$

$$N_W = \frac{R_0}{\pi}\int_0^{u_*} \frac{W(u)}{\rho(u)}\,\mathrm{d}u.$$

We require the nondegeneracy condition

$$N_W > 0.$$

The echo ratio and calibrated functional are

$$Q = \frac{y_e}{y_d}, \qquad d_\alpha = \widehat{Q}(\alpha) = \frac{Q}{N_W}.$$

They are continuum functionals, not yet finite classical outputs. The fixed geometry, pulse, and windows make W, u_*, N_W independent of α.

Condition excludes the zero window and windows having no overlap with the emitted pulse after a diffracted delay. Nonnegativity alone would not exclude those cases.

Lemma (ANALYTIC: exact averaging formula). *For the windows of Definition ,*

$$y_d = \frac{1}{4\pi R_0}.$$

Let

$$q(\alpha, u) = -\frac{\Phi_0(u,0)}{\alpha} = -\frac{\sin(\pi/\alpha)}{\alpha(\cosh(u/\alpha) - \cos(\pi/\alpha))}.$$

Then

$$d_\alpha = \frac{\int_0^{u_*} q(\alpha,u)W(u)\rho(u)^{-1}\,\mathrm{d}u}{\int_0^{u_*} W(u)\rho(u)^{-1}\,\mathrm{d}u}.$$

The denominator is positive. Rescaling the source by a nonzero common amplitude rescales both y_d and y_e and leaves Q unchanged.

Proof. The direct window contains the complete direct pulse, no image pulse, and no diffracted signal. Pulse normalization gives the first formula. The tail margin ensures that the echo window contains no geometric pulse. Apply Fubini's theorem to ; on its relevant compact *u*-range the integrand is bounded after temporal convolution. Also $W(u) = 0$ for $u > u_*$. Dividing the resulting echo integral by $y_d N_W$ gives .

par *Non-extraction and non-sourcing boundary.* The conclusion concerns a continuum functional and, only under the declared preparation assumption, a finite record law. Destructive Enter does not acquire that functional, select the host, or turn a conserved tensor into a source of the cone. No geometry update or Einstein equation is inferred [2, 9, 7, 10].

Deficit witnessing and windowed separation

Set

$$q_0(\alpha) = q(\alpha,0) = -\frac{1}{\alpha}\cot\frac{\pi}{2\alpha}, \qquad D(\alpha) = 1 - \cos\frac{\pi}{\alpha}.$$

On every sector, $D > 0$.

Theorem (ANALYTIC: witness, error, and sector separation). *Let the common windows satisfy Definition . For $\alpha \notin \mathscr{S}$, define*

$$\varepsilon_W(\alpha,u_*) = \frac{|\sin(\pi/\alpha)|(\cosh(u_*/\alpha) - 1)}{\alpha D(\alpha)^2}.$$

Then:

1. *On* $1/2 < \alpha \le 1$,

$$d_1 = 0, \qquad d_\alpha \ge q(\alpha,u_*) > 0 \quad \text{if} 1/2 < \alpha < 1.$$

Thus $d_{\alpha} = 0$ *if and only if* $\mu = 0$*, within this range.*

2. *On every sector,*

$$q_0'(\alpha) \le -\frac{\pi}{2}.$$

3. *For every admissible* α*,*

$$|d_{\alpha} - q_0(\alpha)| \le \varepsilon_W(\alpha, u_*), \qquad |d_{\alpha}| \le |q_0(\alpha)|.$$

4. *For a compact interval* $J \subset \mathscr{I}_m$*, put*

$$E_J = \max_{\alpha \in J} \varepsilon_W(\alpha, u_*).$$

For $\alpha_1, \alpha_2 \in J$*,*

$$|d_{\alpha_1} - d_{\alpha_2}| \ge \frac{\pi}{2}|\alpha_1 - \alpha_2| - 2E_J.$$

Consequently, the averaged functional separates every pair with $|\alpha_1 - \alpha_2| > 4E_J/\pi$*.*

5. *On* $[\alpha_{\min}, 1]$*, with* $\alpha_{\min} > 1/2$*,* $\varepsilon_W(\alpha, u_*)$ *decreases with* α*, so* $E_J \le \varepsilon_W(\alpha_{\min}, u_*)$*.*

Proof. Write $A = \pi/\alpha$ and $H(\alpha, u) = \cosh(u/\alpha) - 1$. Directly from ,

$$q(\alpha, u) = q_0(\alpha)\frac{D(\alpha)}{D(\alpha) + H(\alpha, u)}.$$

For $1/2 < \alpha < 1$, $\sin A < 0$, so $q > 0$ and it decreases in u. At $\alpha = 1$, the untwisted diffraction weight vanishes.

Averaging proves (i), including its strictness because the weight has positive integral.

For (ii), let $x = \pi/(2\alpha)$. Differentiation gives

$$q_0'(\alpha) = -\frac{x - \sin x \cos x}{\alpha^2 \sin^2 x}.$$

The numerator equals $\pi/2$ at $x = \pi/2$ and has derivative $2\sin^2 x \ge 0$. Since $x \ge \pi/2$ and $\alpha^2 \sin^2 x \le 1$, follows

wherever defined.

For (iii), gives

$$|q(\alpha,u)-q_0(\alpha)| = \frac{|\sin A|\,H(\alpha,u)}{\alpha D(\alpha)(D(\alpha)+H(\alpha,u))} \le \varepsilon_W(\alpha,u_*).$$

It also gives $|q(\alpha,u)| \le |q_0(\alpha)|$. Average both inequalities.

The mean value theorem on the interval J, followed by the triangle inequality and (iii), proves (iv).

On $1/2 < \alpha \le 1$,

$$\varepsilon_W(\alpha,u_*) = \frac{|\cos(\pi/(2\alpha))|}{2\alpha\sin^3(\pi/(2\alpha))}(\cosh(u_*/\alpha)-1).$$

As α decreases, each nonnegative factor increases. This proves (v).

par *Non-extraction and non-sourcing boundary.* The conclusion concerns a continuum functional and, only under the declared preparation assumption, a finite record law. Destructive Enter does not acquire that functional, select the host, or turn a conserved tensor into a source of the cone. No geometry update or Einstein equation is inferred [2, 9, 7, 10].

Remark (ANALYTIC: limitation of the witness). The zero-deficit witness is not asserted on the full range. At $\alpha = 1/N$ with $N > 1$ odd, the untwisted kernel is pure image and $d_\alpha = 0$. These deficits are not confused with flat space by the complete geometric signal, but they cannot be distinguished by the diffracted amplitude alone.

Injectivity for a nonzero finite window

The front reference is injective on each sector. The following result also establishes injectivity for sufficiently short *nonzero* windows, rather than identifying a zero-width limit with an acquisition procedure.

Theorem (ANALYTIC: finite-window injectivity on a compact sector). *Let $J = [a,b] \subset \mathscr{I}_m$, and define*

$$d_* = \min_{\alpha\in J} D(\alpha) > 0, \qquad M_0 = \max_{\alpha\in J}|q_0(\alpha)|, \qquad M_1 = \max_{\alpha\in J}|q_0'(\alpha)|.$$

Put

$$L_J(u_*) = u_*^2\cosh(u_*/a)\left[\frac{M_1}{2a^2d_*} + \frac{M_0}{a^3d_*} + \frac{\pi M_0}{2a^4d_*^2}\right].$$

If $L_J(u_*) < \pi/2$, then the averaged functional is strictly decreasing and satisfies

$$|d_{\alpha_1} - d_{\alpha_2}| \geq \left(\frac{\pi}{2} - L_J(u_*)\right)|\alpha_1 - \alpha_2|, \qquad \alpha_1, \alpha_2 \in J.$$

Such a condition is available for sufficiently short windows, provided the pulse/window overlap remains nonzero and the image-isolation conditions remain satisfied.

Proof. From ,

$$q - q_0 = -q_0 \frac{H}{D + H}.$$

For $0 \leq u \leq u_*$ and $\alpha \in J$,

$$H \leq \frac{u^2}{2a^2}\cosh(u_*/a), \qquad |\partial_\alpha H| \leq \frac{u^2}{a^3}\cosh(u_*/a), \qquad |D'| \leq \frac{\pi}{a^2}.$$

Differentiating $q - q_0$, and using $D + H \geq d_*$, gives

$$|\partial_\alpha(q - q_0)| \leq \frac{M_1 H}{d_*} + M_0\left(\frac{|\partial_\alpha H|}{d_*} + \frac{H|D'|}{d_*^2}\right) \leq L_J(u_*).$$

The averaging measure in is independent of α. Differentiation under its integral is valid on the compact parameter set, so

$$d'_\alpha \leq -\frac{\pi}{2} + L_J(u_*).$$

Integration between the two parameters proves the result.

par *Non-extraction and non-sourcing boundary.* The conclusion concerns a continuum functional and, only under the declared preparation assumption, a finite record law. Destructive Enter does not acquire that functional, select the host, or turn a conserved tensor into a source of the cone. No geometry update or Einstein equation is inferred [2, 9, 7, 10].

What remains global and front-level

Proposition (ANALYTIC: full-range front-level identification). *For neutral backscatter with $\alpha \in (0, 1] \setminus \mathscr{S}$, the pair*

$$\big(\text{number of distinct geometric lengths, } c_0(\alpha, 0)\big)$$

determines α.

Proof. The number is $m + 1$ on $\mathscr{I}_m$, by Lemma , so it determines the sector. On that sector,

$$\frac{\mathrm{d}}{\mathrm{d}\alpha}\cot\frac{\pi}{2\alpha} = \frac{\pi}{2\alpha^2}\csc^2\frac{\pi}{2\alpha} > 0.$$

The front coefficient therefore determines at most one parameter within the identified sector.

par *Non-extraction and non-sourcing boundary*. The conclusion concerns a continuum functional and, only under the declared preparation assumption, a finite record law. Destructive Enter does not acquire that functional, select the host, or turn a conserved tensor into a source of the cone. No geometry update or Einstein equation is inferred [2, 9, 7, 10].

Remark (OPEN: no global resolution guarantee). Proposition assumes that distinct geometric lengths and the front coefficient are already available. It is not an instrument for counting unresolved arrivals. There is no uniform pulse-duration margin as a parameter approaches a shadow value, and no uniform lower bound on image-time gaps as $\alpha \mathrm{downarrow}\, 0$. The sector theorems therefore do not constitute one fixed-resolution global separation theorem. At $\alpha \in \mathscr{S}$, the total kernel must be treated without separately evaluating the singular decomposition.

Noise, timing, and admissible smoothing

Proposition (ANALYTIC: record and timing errors). *Suppose*

$$|\hat{y}_d - y_d| \le \eta_d < y_d, \qquad |\hat{y}_e - y_e| \le \eta_e.$$

Then

$$|\widehat{Q} - Q| \le \frac{\eta_e + |Q|\eta_d}{y_d - \eta_d}.$$

With exact N_W, define a common upper bound

$$\eta_Q \ge \frac{\eta_e + |Q|\eta_d}{N_W(y_d - \eta_d)}.$$

For two reported calibrated values in a common sector,

$$|\widetilde{d}_{\alpha_1} - \widetilde{d}_{\alpha_2}| \geq \frac{\pi}{2}|\alpha_1 - \alpha_2| - 2E_J - 2\eta_Q.$$

For unit-height interval gates, endpoint errors of size at most τ_c give

$$\eta_e \leq 2\tau_c \sup_{\mathscr{I}} |u_{\mathrm{dif}}|,$$

provided the affected time intervals $\mathscr{I}$ do not meet a geometric pulse. The direct error is zero if the direct window keeps the required margins around the entire direct pulse.

Proof. Use

$$\widetilde{Q} - Q = \frac{(\widetilde{y}_e - y_e) - Q(\widetilde{y}_d - y_d)}{\widetilde{y}_d}.$$

The denominator has modulus at least $y_d - \eta_d$, proving . The separation estimate follows by two applications of the triangle inequality. A shifted endpoint changes an interval integral by at most τ_c times the supremum of its integrand on the affected interval. There are two endpoints.

par *Non-extraction and non-sourcing boundary.* The conclusion concerns a continuum functional and, only under the declared preparation assumption, a finite record law. Destructive Enter does not acquire that functional, select the host, or turn a conserved tensor into a source of the cone. No geometry update or Einstein equation is inferred [2, 9, 7, 10].

Remark (ANALYTIC: the correct smoothing statement). Nonnegative smoothing preserves the averaging proof only when the resulting pulse and effective window still satisfy the support, isolation, and positive-normalization hypotheses. It then changes W and, if necessary, u_*. An arbitrary smoothing kernel can leak a geometric pulse into the echo gate and destroy the witness, even at $\alpha = 1$. Thus ``nonnegative smoothing" alone is not a sufficient robustness hypothesis. Calibration uncertainty in N_W must also be included in the declared bound on η_Q.

Finite supplied-report instruments

Finite values versus finite outputs

The continuum quantities

$$(y_d, y_e, d_\alpha)$$

are real numbers. They are not exact classical outputs of a finite alphabet. To report them, one must declare finite ranges, precision, rounding conventions, failure labels, and enclosures. On a fixed compact sector these ranges are bounded.

Assumption (SUPPLIED: finite preparation and acquisition interface). For each requested real task value d, the experimental or documentary interface supplies:

1. a finite code for an approximation or enclosure, including failure and out-of-range words;

2. a value $\hat{d}$ satisfying $|\hat{d} - d| \le \eta_d$ on the declared successful preparation class;

3. a finite signal preparation approximating $\tau_\sigma(v\hat{d})$, with complete preparation error at most e_{acq}. Here $\sigma > 0$, $v \in \mathbb{R}$, and

$$p_\sigma(z) = \frac{1 + \tanh(z/\sigma)}{2},$$

$$|e_\sigma(z)\rangle = \sqrt{1 - p_\sigma(z)}|0\rangle + \sqrt{p_\sigma(z)}|1\rangle,$$

$$\tau_\sigma(z) = |e_\sigma(z)\rangle\langle e_\sigma(z)|.$$

The complete comparison includes every declared surviving laboratory register and untouched finite reference.

The physical map from an unknown continuum field to the supplied codes or signal preparations is not constructed. No continuum input representation or field–apparatus coupling is inferred from this assumption.

Preparation from an already known finite code is a finite task. Obtaining that code from an unknown field is the additional OPEN acquisition problem emphasized by X-J prop:noacquisition [7].

Complete destructive reporting

Theorem (ANALYTIC: clock/report channel). *Let $C \cong \mathbb{C}^D$ have an orthonormal calibration basis $\{|\tau_j\rangle\}$. Let A contain every surviving finite register. Install finite words w_j, retaining the index j even if two words coincide. Put*

$$D_j = \langle \tau_j|_C \otimes I_A.$$

Then

$$\mathscr{M}(X) = \sum_j |j, w_j\rangle\langle j, w_j|_R \otimes D_j X D_j^\dagger$$

is CPTP and

$$\mathrm{Tr}_R\,[(\mathscr{M} \otimes \mathrm{id}_E)(X)] = \mathrm{Tr}_C X$$

for every untouched finite reference E. For a state ρ, the j-block is

$$\sigma_j = D_j \rho D_j^\dagger, \qquad p_j = \mathrm{Tr}\,\sigma_j.$$

Only a positive-probability block is normalized. The returned quantum interface excludes C.

Proof. The recorded Kraus maps are $L_j = |j, w_j\rangle \otimes D_j$, and

$$\sum_j L_j^\dagger L_j = \sum_j |\tau_j\rangle\langle\tau_j| \otimes I_A = I_{CA}.$$

Thus the map is CPTP. Expanding $X = \sum_{j,k} |\tau_j\rangle\langle\tau_k| \otimes X_{jk}$ shows that its output blocks are X_{jj}. Their sum is $\mathrm{Tr}_C X$. Tensoring this calculation with I_E proves the complete statement.

This is the complete supplied-report identity of I-B and En09-A, also used in I-B-G, I-B-VP-5, and XVI-O1-ST [2, 9, 3, 5, 11]. It is not a new characterization of measurement.

The complete X-J signal channel

Theorem (ANALYTIC: signal channel and Lipschitz bound). *Let $Q \cong \mathbb{C}^2$, $0 \le r < 1$, and let A survive. Define*

$$\mathscr{I}_F(X) = r\,\mathrm{Tr}_Q X,$$

$$\mathscr{I}_{+}(X) = (1 - r)(\langle 1| \otimes I_A)X(|1\rangle \otimes I_A),$$

$$\mathscr{I}_{-}(X) = (1 - r)(\langle 0| \otimes I_A)X(|0\rangle \otimes I_A).$$

The recorded channel

$$\mathscr{J}(X) = \sum_{a \in \{F,+,-\}} \mathrm{proj}\, a \otimes \mathscr{I}_a(X)$$

is CPTP, consumes Q, and obeys

$$\mathrm{Tr}_{\{F,+,-\}}\mathscr{J}(X) = \mathrm{Tr}_Q X,$$

including every untouched reference.

On input $\tau_\sigma(vd)$*, its law is*

$$P_d(F) = r, \qquad P_d(\pm) = \frac{1 - r}{2}\left(1 \pm \tanh\frac{vd}{\sigma}\right).$$

Furthermore,

$$d_{\mathrm{tr}}(\tau_\sigma(z), \tau_\sigma(z')) \le \frac{|z - z'|}{2\sigma}, \qquad d_{\mathrm{tr}}(\rho, \rho') = \frac{1}{2}\|\rho - \rho'\|_1.$$

Proof. Each block in is CP. Summing the blocks gives $\mathrm{Tr}_Q X$, which proves trace preservation and the discard identity. The same calculation holds with a reference. The probabilities follow from .

Write

$$|e_\sigma(z)\rangle = \cos\theta(z)|0\rangle + \sin\theta(z)|1\rangle, \qquad 0 < \theta(z) < \pi/2.$$

Differentiation gives $\theta'(z) = \mathrm{sech}\,(z/\sigma)/(2\sigma)$. The trace distance between these pure states is $|\sin(\theta(z) - \theta(z'))|$, bounded by $|z - z'|/(2\sigma)$.

The effects are

$$E_F = rI, \qquad E_+ = (1 - r)\mathrm{proj}\, 1, \qquad E_- = (1 - r)\mathrm{proj}\, 0.$$

They act on the supplied qubit, not on a continuum field [7]. Standard instrument language and complete channel

comparisons are described in [26, 27].

Proposition (ANALYTIC: finite history assembly). *Apply Theorem to a finite setting/report register and Theorem to supplied signal registers for y_d, y_e, d_α, using distinct consumed names and retaining all declared residuals and failure symbols. Their sequential composition is a complete finite CPTP history channel.*

Its classical output is a finite word together with three symbols in $\{F, +, -\}$, not the exact tuple (y_d, y_e, d_α). Discarding all records leaves the appropriate partial trace over the consumed factors.

If stage ℓ has complete error at most δ_ℓ, in the normalized diamond metric

$$d_\diamond(\mathscr{E}, \mathscr{F}) = \frac{1}{2}\|\mathscr{E} - \mathscr{F}\|_\diamond,$$

the complete history error is at most $\sum_\ell \delta_\ell$.

Proof. Composition of CPTP maps is CPTP, including after adjoining a reference. The discard identities compose. The output alphabets and the number of stages are finite.

For the error bound, subtract the two composed channels and telescope, replacing one stage at a time. Pre- and post-composition by a CPTP map do not increase the diamond norm. The triangle inequality gives the sum bound.

par *Non-extraction and non-sourcing boundary.* The conclusion concerns a continuum functional and, only under the declared preparation assumption, a finite record law. Destructive Enter does not acquire that functional, select the host, or turn a conserved tensor into a source of the cone. No geometry update or Einstein equation is inferred [2, 9, 7, 10].

The proposition realizes the channel identities used in X-K [8]. It does not supply a physical causal embedding, hardware resource certificate, or microscopic detector for the continuum task. Finite clock labels name the declared windows; they do not construct the window integrals.

Instrumented deficit separation

Theorem (ANALYTIC: complete record-law separation). *Fix $J \subset \mathscr{I}_m$, admissible common windows, and the*

calibrated task d_α. Let

$$B_J = \max_{\alpha \in J} |q_0(\alpha)|, \qquad e = e_{\mathrm{acq}} + \frac{|v|\eta_Q}{2\sigma} + e_{\mathrm{comp}}.$$

where in this formula and below $|v|\eta_Q|$ means $|v|\eta_Q$. Equivalently, and without this shorthand, set

$$e = e_{\mathrm{acq}} + \frac{|v|\eta_Q}{2\sigma} + e_{\mathrm{comp}}.$$

Assume $v \neq 0$, Assumption with $|\tilde{d}_\alpha - d_\alpha| \le \eta_Q$, and the complete composition-error bound e_{comp}. The implemented signal marginals satisfy

$$\mathrm{TV}\,(\tilde{P}_{\alpha_1}, \tilde{P}_{\alpha_2}) \ge [\frac{(1-r)|v|}{2\sigma}\mathrm{sech}^2\left(\frac{|v\mathrm{rvertB}_J}{\sigma}\right)$$

$$\times[\frac{\pi}{2}|\alpha_1 - \alpha_2| - 2E_J]_+ - 2e]_+.$$

The full recorded history has at least this total-variation gap.

If Theorem applies, its stronger separation constant may replace the bracketed continuum separation in .

For $1/2 < \alpha < 1$, comparison with $\alpha = 1$ gives

$$\mathrm{TV}\,(\tilde{P}_\alpha, \tilde{P}_1) \ge \left[\frac{1-r}{2}\tanh\left(\frac{|v\mathrm{rvertq}\,(\alpha, u_*)}{\sigma}\right) - 2e\right]_+.$$

Proof. For two ideal inputs, gives exactly

$$\mathrm{TV}\,(P_{d_1}, P_{d_2}) = \frac{1-r}{2}\left|\tanh\frac{vd_1}{\sigma} - \tanh\frac{vd_2}{\sigma}\right|.$$

Since $|d_j| \le B_J$, the mean value theorem gives

$$\mathrm{TV}\,(P_{d_1}, P_{d_2}) \ge \frac{(1-r)|v|}{2\sigma}\mathrm{sech}^2\left(\frac{|v\mathrm{rvertB}_J}{\sigma}\right)|d_1 - d_2|.$$

Apply Theorem , or Theorem .

By , the value error contributes at most $|v|\eta_Q/(2\sigma)$ to the trace-distance error. The residual preparation and complete composition errors add the other terms in . Measurement is contractive, so each implemented law is within *e* of its

ideal law. Two applications of the triangle inequality subtract 2*e*. Taking the positive part is legitimate because total variation is nonnegative.

Marginalization cannot increase total variation, so the full history has at least the signal-marginal gap. Finally $d_1 = 0$ for the flat untwisted channel and $d_\alpha \geq q(\alpha, u_*) > 0$, proving .

par *Non-extraction and non-sourcing boundary.* The conclusion concerns a continuum functional and, only under the declared preparation assumption, a finite record law. Destructive Enter does not acquire that functional, select the host, or turn a conserved tensor into a source of the cone. No geometry update or Einstein equation is inferred [2, 9, 7, 10].

Remark (ANALYTIC: statistical meaning). A positive gap is a distinction between probability laws under the stated preparation and error hypotheses. It is not exact identification of an arbitrary real deficit from one finite outcome. Neither the number of repetitions nor an empirical noise distribution is fixed by this theorem.

Holonomy channels and the higher-spin boundary

Proposition (ANALYTIC: twisted-scalar front information). *For* $1/2 < \alpha \leq 1$, *define* $\delta = |\beta - \frac{1}{2}| \in [0, \frac{1}{2}]$. *Then*

$$c_0(\alpha, \beta) = \frac{\cos(\pi\delta/\alpha)}{\sin(\pi/(2\alpha))}.$$

For known α, this is strictly decreasing in δ. It determines the folded holonomy parameter δ, not the orientation β versus $1 - \beta$.

At $\alpha = 1$,

$$|c_0(1, \beta_1) - c_0(1, \beta_2)| \geq 2|\delta_1^2 - \delta_2^2|.$$

At a nonzero non-shadow angle, $\Phi_{1-\beta}(0, \theta) = \overline{\Phi_\beta(0, \theta)}$. *A calibrated complex phase comparison can distinguish these two possibilities when the coefficient is nonreal. Each geometric image carries the relative bundle factor* $e^{2\pi i\beta k}$.

Proof. Equation is . Its denominator is positive, and $\pi\delta/\alpha \in [0, \pi)$, where cosine is strictly decreasing. For $\alpha = 1$, assume $\delta_1 \le \delta_2$ and use

$$\cos(\pi\delta_1) - \cos(\pi\delta_2) = 2\sin\frac{\pi(\delta_1 + \delta_2)}{2}\sin\frac{\pi(\delta_2 - \delta_1)}{2}.$$

Both sine arguments lie in $[0, \pi/2]$. The inequality $\sin x \ge 2x/\pi$ gives $2(\delta_2^2 - \delta_1^2)$. The remaining assertions follow from and .

par *Non-extraction and non-sourcing boundary.* The conclusion concerns a continuum functional and, only under the declared preparation assumption, a finite record law. Destructive Enter does not acquire that functional, select the host, or turn a conserved tensor into a source of the cone. No geometry update or Einstein equation is inferred [2, 9, 7, 10].

Remark (OPEN: phase acquisition). A complex amplitude is not an input to a real scalar signal task without further specification. One may declare separate real and imaginary tasks with a common phase reference, but their acquisition, calibration, and error budgets are additional instances of Assumption . Backscatter front monotonicity alone is not a finite-window inversion theorem for the charged channel.

Input statement (SUPPLIED: spin-1 channel convention). In the compatible spin-1 setting of I-B-G-S, use parallel covariant components of spin weight $\sigma = 0, \pm 1$, with

$$\beta_\sigma \equiv \sigma\alpha \bmod 1.$$

The relevant potential Green operator is channel diagonal with the componentwise Friedrichs scalar blocks. Gauge interpretation requires the source, test, and domain compatibility hypotheses of that manuscript, including conserved sources where specified [4].

Proposition (ANALYTIC: spin-1 front and image identities). *Under Input statement , the two transverse front weights obey*

$$\Phi_\alpha(0, \theta) = -e^{i\theta}\Phi_0(0, \theta), \qquad \Phi_{1-\alpha}(0, \theta) = -e^{-i\theta}\Phi_0(0, \theta).$$

At $\alpha = 1$, the twists are interpreted modulo one. Thus the helicity moduli agree at the front.

For the k-th image, the spin-weight factor is

$$e^{2\pi i\sigma\alpha k} = e^{-i\sigma\mu k}.$$

In the declared developed transverse frame, these two characters represent the rotation by $-k\mu$.

Proof. For $\beta = \alpha$ in ,

$$(\beta - \tfrac{1}{2})\xi = \theta - \xi/2, \qquad (\beta - \tfrac{1}{2})a = \pi - a/2.$$

Comparison with the $\beta = 0$ formula gives the first identity. Complex conjugation gives the second. The image factors follow directly from the channel twists and $2\pi\alpha = 2\pi - \mu$.

par *Non-extraction and non-sourcing boundary.* The conclusion concerns a continuum functional and, only under the declared preparation assumption, a finite record law. Destructive Enter does not acquire that functional, select the host, or turn a conserved tensor into a source of the cone. No geometry update or Einstein equation is inferred [2, 9, 7, 10].

Remark (ANALYTIC: component weights are not a detector). Equation is an identity for the specified potential-kernel blocks. A gauge-invariant photon observable requires an appropriate source/test contraction or field-strength operation. Equal helicity moduli at this kernel front do not, by themselves, prove an apparatus-level polarization theorem for arbitrary sources and receivers.

Core extensions: the missing data

Proposition (ANALYTIC: obstruction to a flat twisted filling). *A smooth flat $U(1)$ connection on a disk has trivial holonomy around every loop bounding a subdisk. Consequently, a nontrivial exterior scalar holonomy cannot be filled by a smooth, everywhere flat connection on a disk.*

Proof. On a disk the bundle is trivializable, and a flat connection has curvature $\mathrm{d}A = 0$. Stokes' theorem gives $\exp(\mathrm{i}\oint A) = \exp(\mathrm{i}\int \mathrm{d}A) = 1$.

OPEN: twisted scalar cores. A smooth filling with nontrivial exterior holonomy must supply interior curvature or flux, or retain a puncture. The interior connection, covariant form, matching data, and limiting extension must therefore be specified. The neutral scalar proof below does not supply them.

OPEN: spin-1 cores. The scalar operator $-\Delta + \xi R$ is not automatically the gauge-compatible vector operator on a curved core. A vector family must specify its bundle, curvature terms, domains, source conservation, and gauge intertwiners. The cone channel identity does not prove convergence of that family.

OPEN: spin-2 cores and curvature-regular front. The curvature-regular spin-2 branch of I-B-G-S is not obtained merely by substituting $\beta = \{2\alpha\}$ into the scalar Friedrichs formula. Its channel domains and any non-Friedrichs corrections must be retained. No curvature-regular spin-2 front coefficient or matter-core selection theorem is released here [4].

These are explicit residuals, not constructed new Green operators. Scalar apex contacts, spin-2 boundary parameters, and normalization choices for an action Hessian are not identified.

Neutral smooth cores and finite-precision indistinguishability

Fix $0 < \alpha < 1$ and a curvature coupling $\zeta \geq 0$. Use ζ, rather than ξ, to avoid conflict with the angular abbreviation in Section .

Definition (Restricted neutral core class). For each $\varepsilon > 0$, let p_ε be smooth and satisfy

$$\alpha \leq p_\varepsilon \leq 1, \qquad p_\varepsilon' \geq 0, \qquad p_\varepsilon = \alpha \text{near} r = 0, \qquad p_\varepsilon = 1 \text{for} r \geq \varepsilon.$$

The filled spatial metric and operator are

$$q_\varepsilon = p_\varepsilon(r)^2 \mathrm{d}r^2 + \alpha^2 r^2 \mathrm{d}\varphi^2 + \mathrm{d}z^2, \qquad B_\varepsilon = -\Delta_{q_\varepsilon} + \zeta R_\varepsilon,$$

defined by their nonnegative closed quadratic forms. Near the axis the metric is smooth in proper Cartesian coordinates. Its scalar curvature is

$$R_\varepsilon = \frac{2p_\varepsilon'}{r p_\varepsilon^3} \geq 0.$$

Let $B_0 = A_{\mathrm{F},0}$ be the neutral cone operator.

This is the scalar nonnegative core class relevant to I-B-VP-5 [5]. The profiles need not have uniformly bounded derivatives as εdownarrow 0.

The Hilbert spaces are

$$\mathscr{H}_\varepsilon = L^2(\mathrm{d}V_\varepsilon), \qquad \mathrm{d}V_\varepsilon = p_\varepsilon \mathrm{d}V_0, \qquad U_\varepsilon : \mathscr{H}_\varepsilon \to \mathscr{H}_0, \quad U_\varepsilon v = \sqrt{p_\varepsilon}\, v.$$

Set $\widetilde{B}_\varepsilon = U_\varepsilon B_\varepsilon U_\varepsilon^{-1}$.

Lemma (ANALYTIC: neutral form selection). *For every family in Definition ,*

$$\widetilde{B}_\varepsilon \to B_0 \quad \text{in the strong-resolvent sense.}$$

Proof. We verify the weak lower-bound and strong recovery properties for the transferred closed forms.

First, on the neutral cone, the Friedrichs form domain is precisely the space of L^2 functions with finite weak Dirichlet energy on the regular region. The axis has zero local H^1-capacity. For completeness, for $0 < \delta < 1$, choose a radial cutoff which is zero on $r \leq \delta^2$, one on $r \geq \delta$, and has derivative bounded by $C/(r|\log \delta|)$ between them. On a fixed compact z-range,

$$\int |\nabla \chi_\delta|^2 \, \mathrm{d}V_0 \leq \frac{C'}{|\log \delta|} \to 0.$$

For bounded compactly supported finite-energy functions, multiplication by this cutoff approximates in form norm. Truncation, compact cutoffs, and smooth approximation away from the axis give the general case. This proves the stated characterization of the Friedrichs form.

Suppose w_ε rightharpoonup w in $\mathscr{H}_0$, with bounded transferred energies. On every fixed exterior region $r > \delta$, for $\varepsilon < \delta$ one has $p_\varepsilon = 1$, $U_\varepsilon = 1$, and $R_\varepsilon = 0$. Weak compactness in local H^1, followed by lower semicontinuity, gives

the cone-energy lower bound on that region. Letting $\delta \mathrm{downarrow}\, 0$, and using nonnegativity of the remaining core energy, gives

$$\mathfrak{b}_0[w] \le \liminf_{\varepsilon \mathrm{downarrow}\, 0} \tilde{\mathfrak{b}}_\varepsilon[w_\varepsilon].$$

The preceding capacity argument places w in the Friedrichs domain.

Conversely, approximate w in cone form norm by $w_n \in C_c^\infty$ supported away from the axis. For all sufficiently small ε, the transferred form on w_n equals the cone form exactly. A diagonal choice $n = n(\varepsilon)$ provides strong recovery.

These are the Mosco form-convergence conditions; their strong-resolvent consequence gives [23, 24].

Smeared records

Let $h_e, h_r \in C_c^\infty$ be fixed exterior profiles satisfying

$$\operatorname{supp} h_e \subset \{r \ge r_e - s\}, \qquad \operatorname{supp} h_r \subset \{r \ge r_r - s\},$$

with positive $r_e - s, r_r - s$. For sufficiently small cores their L^2 norms and Hilbert identifications are the same as on the cone. Define

$$S_\varepsilon(t) = \frac{\sin(t\sqrt{B_\varepsilon})}{\sqrt{B_\varepsilon}},$$

$$Y^\varepsilon(t) = \int_0^t f(t')\langle h_r, S_\varepsilon(t - t')h_e\rangle \, \mathrm{d}t',$$

$$y^\varepsilon[\chi] = \int_0^T \chi(t) Y^\varepsilon(t) \, \mathrm{d}t, \qquad \chi \in L^1([0, T]).$$

Use B_0 for the cone quantities.

Theorem (ANALYTIC: core agreement, convergence, and bound). *Assume* $\varepsilon + s < \min(r_e, r_r)$, *and put*

$$t_\varepsilon = r_e + r_r - 2\varepsilon - 2s.$$

Then:

1. $Y^\varepsilon(t) = Y^0(t)$ *for* $0 \le t < t_\varepsilon$. *Thus* $y^\varepsilon[\chi] = y^0[\chi]$ *if* $\operatorname{supp}\chi \subset [0, t_\varepsilon)$.

2. *$Y^{\varepsilon} \to Y^{0}$ uniformly on every finite interval $[0,T]$. Consequently,*

$$\sup_{\|\chi\|_1 \le 1} |y^{\varepsilon}[\chi] - y^{0}[\chi]| \to 0.$$

3. *For every ε,*

$$|y^{\varepsilon}[\chi] - y^{0}[\chi]| \le 2T\|f\|_1 \|h_r\| \|h_e\| \int_{t \ge t_\varepsilon} |\chi(t)|\,\mathrm{d}t,$$

where $\|f\|_1$ denotes $\|f\|_{L^1}$.

Proof. For (i), use finite propagation and local uniqueness in the common smooth exterior. More explicitly, choose a separating cylinder $r = b > \varepsilon$, below both profile supports. No wave from the emitter reaches that cylinder before $r_e - s - b$. Any later difference in boundary data on the cylinder requires at least another $r_r - s - b$ to reach the receiver. The two exterior equations and their vanishing past agree. Their receiver responses therefore agree for $t < r_e + r_r - 2b - 2s$. Let $b$downarrow ε. The cone side uses the Friedrichs finite-propagation input.

For (ii), set

$$\varphi_\tau(\lambda) = \begin{cases} \sin(\tau\sqrt{\lambda})/\sqrt{\lambda}, & \lambda > 0, \\ \tau, & \lambda = 0. \end{cases}$$

For fixed τ, this function is continuous, bounded by τ on $[0,\infty)$, and tends to zero as $\lambda \to \infty$. Strong-resolvent convergence gives strong convergence of the corresponding operators [24]. Since $U_\varepsilon h_j = h_j$ on the fixed supports for small ε, dominated convergence gives pointwise convergence of Y^{ε}.

Also

$$\|S_\varepsilon(\tau)\| \le \tau, \qquad \|\partial_\tau S_\varepsilon(\tau)\| \le 1.$$

Hence the functions Y^{ε} are uniformly bounded and Lipschitz on $[0,T]$, with Lipschitz constant at most $\|f\|_{L^1}\|h_r\|\|h_e\|$. Equicontinuity upgrades pointwise convergence to uniform convergence. The L^1-window statement follows immediately.

Finally, each response has modulus at most $T\|f\|_{L^1}\|h_r\|\|h_e\|$. Subtract, use (i), and integrate against $|\chi|$ to prove (iii).

par *Non-extraction and non-sourcing boundary*. The conclusion concerns a continuum functional and, only under the declared preparation assumption, a finite record law. Destructive Enter does not acquire that functional, select the host, or turn a conserved tensor into a source of the cone. No geometry update or Einstein equation is inferred [2, 9, 7, 10].

Corollary (ANALYTIC: finite-precision non-reconstruction). *Fix finitely many smeared windows and any precision $\eta > 0$. For every core family in Definition , all sufficiently small cores reproduce all the cone records within η. These filled metrics are not isometric to the singular cone.*

If signal preparations depend continuously on these finitely many values through , their ideal finite signal laws converge to the cone laws. For any fixed number n of independent repetitions, the total-variation distance of the repeated signal laws also tends to zero.

Thus no such fixed finite-precision protocol uniformly distinguishes the ideal cone from all sufficiently small admissible cores. This is not a proof that the corresponding map of finitely many exact real functionals is non-injective.

Proof. The first assertion follows by applying Theorem (ii) to the finite list. Since $0 < \alpha < 1$, every admissible transition from $p_\varepsilon = \alpha$ to $p_\varepsilon = 1$ has $p_\varepsilon' \equiv 0$. Equation then gives nonzero curvature somewhere, unlike the regular cone exterior; the filled smooth axis also differs from the cone singularity.

Continuity of the signal laws follows from . For product laws,

$$\mathrm{TV}\,(P_\varepsilon^{\otimes n}, P_0^{\otimes n}) \le n\,\mathrm{TV}\,(P_\varepsilon, P_0),$$

by telescoping the tensor products. The final qualification follows because arbitrarily close values need not be equal values.

par *Non-extraction and non-sourcing boundary*. The conclusion concerns a continuum functional and, only under the declared preparation assumption, a finite record law. Destructive Enter does not acquire that functional, select the host, or turn a conserved tensor into a source of the cone. No geometry update or Einstein equation is inferred

[2, 9, 7, 10].

Remark (ANALYTIC: additional qualifications). The diffraction window begins after the exact agreement interval, so it is core-sensitive at finite ε. The theorem gives convergence, not equality, there. It concerns fixed spatially smeared profiles and does not justify a point-source or point-receiver limit.

Ratios also converge whenever their limiting denominator is nonzero, by continuity of division. The explicit point-receiver deficit constants of Section are not thereby transferred to a spatially smeared experiment.

A separate finite report code can jump at a rounding boundary. Convergence of its deterministic word requires a margin from that boundary; convergence of the continuous signal law does not.

A quantitative core-error interface and the missing rate

Resolvent discrepancy controls a windowed record

For $a > 0$, $s = a + \mathrm{i}\omega$, define

$$\mathscr{R}_\varepsilon(s) = (\widetilde{B}_\varepsilon + s^2)^{-1}, \qquad \Delta_\varepsilon(s) = \langle h_r, (\mathscr{R}_\varepsilon(s) - \mathscr{R}_0(s))h_e\rangle.$$

For $\Omega > 0$, put

$$\mathscr{E}_\varepsilon(a, \Omega) = \sup_{|\omega| \le \Omega} |\Delta_\varepsilon(a + \mathrm{i}\omega)|.$$

Let

$$\hat{f}_a(\omega) = \int_0^\infty e^{-(a + \mathrm{i}\omega)t} f(t)\, \mathrm{d}t, \qquad F_{N,a} = \|(\partial_t^N (e^{-at} f(t)))\|_{L^1(\mathbb{R})}.$$

Theorem (ANALYTIC: explicit resolvent-to-record bound). *For $\chi \in L^1([0, T])$, $a > 0$, $\Omega > 0$, and an integer $N \ge 2$,*

$$|y^\varepsilon[\chi] - y^0[\chi]| \le e^{aT}\|\chi\|_1 \Big[\frac{\mathscr{E}_\varepsilon(a, \Omega)}{2\pi} \int_{-\Omega}^{\Omega} |\hat{f}_a(\omega)|\, \mathrm{d}\omega + \frac{2\|h_r\|\|h_e\| \mathrm{rVert} F_{N,a}}{\pi a^2 (N - 1)} \Omega^{1 - N}\Big].$$

Here $\|\chi\|_1 = \|\chi\|_{L^1}$. *For fixed* a, Ω, $\mathscr{E}_\varepsilon(a,\Omega) \to 0$. *An explicit bound on this quantity therefore supplies a quantitative record estimate, including after* t_ε.

Proof. The spectral theorem and the Laplace transform of the sine propagator give

$$\int_0^\infty e^{-st} S_\varepsilon(t)\,\mathrm{d}t = (\widetilde{B}_\varepsilon + s^2)^{-1}, \qquad \operatorname{Re} s > 0.$$

The bound $\|S_\varepsilon(t)\| \le t$ yields

$$\|\mathscr{R}_\varepsilon(a + \mathrm{i}\omega)\| \le a^{-2}, \qquad |\Delta_\varepsilon(a + \mathrm{i}\omega)| \le 2a^{-2}\|h_r\|\|h_e\|.$$

Fourier inversion of the exponentially damped response gives

$$Y^\varepsilon(t) - Y^0(t) = \frac{e^{at}}{2\pi}\int_{\mathbb{R}} e^{\mathrm{i}\omega t}\widehat{f}_a(\omega)\Delta_\varepsilon(a + \mathrm{i}\omega)\,\mathrm{d}\omega.$$

The integral is absolutely convergent because f is smooth and compactly supported.

Split the integral at $|\omega| = \Omega$, then integrate against $|\chi|$. Repeated integration by parts gives

$$|\widehat{f}_a(\omega)| \le F_{N,a}|\omega|^{-N}, \qquad \int_{|\omega| > \Omega} |\widehat{f}_a(\omega)|\,\mathrm{d}\omega \le \frac{2F_{N,a}}{N-1}\Omega^{1-N}.$$

These estimates prove .

Strong-resolvent convergence gives pointwise convergence of the matrix elements. On a compact ω-interval, the resolvent identity and the common bound a^{-2} give a uniform Lipschitz bound in ω. Equicontinuity then makes the convergence uniform, proving the last assertion.

par *Non-extraction and non-sourcing boundary.* The conclusion concerns a continuum functional and, only under the declared preparation assumption, a finite record law. Destructive Enter does not acquire that functional, select the host, or turn a conserved tensor into a source of the cone. No geometry update or Einstein equation is inferred [2, 9, 7, 10].

Exterior mode matching

The following calculation identifies the profile-dependent information needed by Theorem . It is the elementary separated-variable matching interface associated with the I-B-VP-5-Ex1 programme record [6]. Its normalization is derived here.

For $m \in \mathbb{Z}$, $k \in \mathbb{R}$, and $s = a + \mathrm{i}\omega$, set

$$\nu_m = \frac{|m|}{\alpha}, \qquad \kappa = \sqrt{k^2 + s^2}, \qquad \operatorname{Re}\kappa > 0.$$

The square root is unambiguous because $a > 0$. Let $v_{\varepsilon,m}(r;\kappa)$ be a nonzero regular interior solution of

$$-\frac{1}{p_\varepsilon r}\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{r}{p_\varepsilon}\frac{\mathrm{d}v}{\mathrm{d}r}\right) + \frac{m^2}{\alpha^2 r^2}v + \zeta R_\varepsilon v + \kappa^2 v = 0.$$

Near the smooth axis it has the regular behavior $r^{|m|}$ for $m \neq 0$, and the regular constant branch for $m = 0$. Its normalization will cancel.

Put $x = \kappa\varepsilon$, $V = v_{\varepsilon,m}(\varepsilon;\kappa)$, $V' = v'_{\varepsilon,m}(\varepsilon;\kappa)$, and define

$$C_{\varepsilon,m}(\kappa) = \frac{xI'_{\nu_m}(x)V - \varepsilon V' I_{\nu_m}(x)}{\varepsilon V' K_{\nu_m}(x) - xVK'_{\nu_m}(x)}.$$

Proposition (ANALYTIC: exterior correction coefficient). *For exterior points $r, r' > \varepsilon$, the separated radial resolvent differs from the cone Friedrichs radial resolvent by*

$$C_{\varepsilon,m}(\kappa)K_{\nu_m}(\kappa r)K_{\nu_m}(\kappa r').$$

Accordingly the full exterior kernel difference is

$$\frac{1}{4\pi^2\alpha}\sum_{m\in\mathbb{Z}} e^{\mathrm{i}m(\varphi-\varphi')}\int_{\mathbb{R}} e^{\mathrm{i}k(z-z')}C_{\varepsilon,m}(\kappa)K_{\nu_m}(\kappa r)K_{\nu_m}(\kappa r')\,\mathrm{d}k,$$

understood first through separated spectral truncation and exterior testing. Absolute rearrangement requires an integrable majorant.

Proof. In the exterior, the radial equation is the modified Bessel equation. A solution matching the regular interior

branch is proportional to

$$I_{\nu_m}(\kappa r) + CK_{\nu_m}(\kappa r).$$

Matching its value and radial derivative at $r = \varepsilon$ gives . Since $p_\varepsilon = 1$ there, the flux matching is precisely this derivative matching.

The decaying exterior solution is $K_{\nu_m}(\kappa r)$. The Wronskian identity

$$I_\nu(\kappa r)\partial_r K_\nu(\kappa r) - \partial_r I_\nu(\kappa r)K_\nu(\kappa r) = -\frac{1}{r}$$

shows that the radial Green kernel relative to $r\,\mathrm{d}r$ is

$$(I_\nu(\kappa r_<) + CK_\nu(\kappa r_<))K_\nu(\kappa r_>).$$

Subtract its $C = 0$ cone counterpart to obtain .

The denominator in cannot vanish for the present resolvent parameters. Otherwise a regular interior solution would match a decaying exterior solution, producing an eigenfunction at transverse spectral value $-\kappa^2$. This is incompatible with self-adjointness and nonnegativity: that value is nonreal when $\omega \neq 0$, and negative when $\omega = 0$.

Angular and longitudinal Fourier inversion contribute the factors $(2\pi\alpha)^{-1}$ and $(2\pi)^{-1}$, respectively. This proves .

For fixed exterior profiles, define

$$A_{r,m}(s,k) = \int \overline{h_r(r,\varphi,z)} e^{\mathrm{i}m\varphi + \mathrm{i}kz} K_{\nu_m}(\kappa r)\,\mathrm{d}V_0,$$

$$A_{e,m}(s,k) = \int h_e(r,\varphi,z) e^{-\mathrm{i}m\varphi - \mathrm{i}kz} K_{\nu_m}(\kappa r)\,\mathrm{d}V_0.$$

Whenever the following quantity is finite,

$$\mathscr{M}_\varepsilon(a,\Omega) = \frac{1}{4\pi^2\alpha} \sup_{|\omega| \le \Omega} \sum_m \int_{\mathbb{R}} |C_{\varepsilon,m}(\kappa) A_{r,m}(s,k) A_{e,m}(s,k)|\,\mathrm{d}k,$$

Proposition gives $\mathscr{E}_\varepsilon(a,\Omega) \le \mathscr{M}_\varepsilon(a,\Omega)$. Substitution into is a quantitative, profile-dependent estimate.

OPEN: the explicit decay rate. The missing estimate is a controlled bound on $C_{\varepsilon,m}$, with the mode,

longitudinal-frequency, and complex-frequency summation needed in . A low-energy estimate for one real-frequency mode is not that bound. No interchange of an ε-limit with the full sum is made without domination. Core sensitivity and logarithmic matching parameters have a substantial history [21, 22]; those results are not substituted for the required time-window estimate.

Remark (ANALYTIC: why strong-resolvent convergence gives no rate). Strong-resolvent convergence alone has no prescribed numerical modulus. For example, on a one-dimensional Hilbert space, $B_{\varepsilon} = c_{\varepsilon} I$ converges strongly in resolvent to zero whenever c_{ε}downarrow 0, and c_{ε} may tend to zero arbitrarily slowly. This example does not prove a slow-rate counterexample within Definition . It shows only why an explicit rate cannot be inferred from the abstract convergence theorem without further quantitative information.

Numerical integrity and retained reports

SUPPLIED provenance; UNCERTIFIED accuracy. The original I-B-ECHO source reports scripts in code/ and logs in logs/. Those artifacts were not supplied as executable attachments to this revision and have not been rerun here. The tables below preserve the reported values. They are not proof premises, error certificates, or claims of independently reproduced accuracy.

The integrity convention is the one used in XVI-O1-ST: a residual is retained together with its provenance, reference, normalization, and missing error sources [11]. A small residual at finitely many points is not a distributional identity or exhaustive parameter coverage [12].

begingroup toprule Reported comparison Retained report Script identifier

midrule endfirsthead toprule Reported comparison Retained report Script identifier

midrule endhead bottomrule endfoot Closed weight versus mode sum Maximum absolute discrepancy 6.0×10^{-31} check_kernel

Scalar front formula 8.7×10^{-30} check_kernel

Pure images, $N = 2, \ldots, 5$, $\beta \in \{0, 1/N\}$ Maximum $|\Phi| = 1.2 \times 10^{-30}$ check_kernel

AB front at $\alpha = 1\ 9.9 \times 10^{-32}$ check_kernel

Twisted backscatter, 20 parameter pairs 1.3×10^{-25} spin_ab

Spin-1 front identity 3.3×10^{-30} identities

Evenness; conjugation 2.1×10^{-30}; 1.1×10^{-30} identities

Laplace kernel versus Bessel/Kontorovich resolvent, four cases including $\beta = 0.3, 0.25$ Relative discrepancies 1.2×10^{-17}–3.9×10^{-14} laplace_bessel

Opposite twist sign in the preceding comparison Discrepancies 4×10^{-4}–6×10^{-3} laplace_bessel

Frequency-domain echo divided by nonzero GTD leading term $k|\text{ratio} - 1| \to 0.916$ at $(\alpha, \theta) = (0.8, 0)$; 2.585 at $(0.7, 0.42)$, $k \le 400$ gtd_freq

Readout grid, $\alpha = 0.55, \ldots, 1$, step 0.01 Reported decreasing grid; minimum calibrated secant magnitude 1.6016; minimum uncalibrated magnitude 0.01608 readout_grid

Window-error ratio on that grid Maximum $|\widehat{Q} - q_0|/\varepsilon_W = 0.252$ readout_grid

Flat-cone numerical readout $Q(1) = -1.1 \times 10^{-34}$; analytic value is exactly zero readout_grid

Flat diffracted waveform Reported maximum $|u_{\text{dif}}| = 10^{-29}$; analytic value is exactly zero waveforms

Pure-image waveform at $\alpha = 1/3$ Reported $|\Phi| \sim 10^{-14}$; analytic weight vanishes waveforms

Nested Euclidean-kernel Laplace calculation Stopped as too slow; no result released. The source calls the Bessel check its replacement check_kernel

endgroup

Remark (SUPPLIED correction to the numerical summary). The old abstract's phrase ``ratio $\le 0.25$'' is not retained as an exact bound: the supplied grid maximum is 0.252. ``About one quarter'' is only a descriptive

rounding. The analytic theorem proves a ratio at most one when $\varepsilon_W > 0$, independently of that grid. At a zero numerator and zero error bound the quotient is not evaluated.

SUPPLIED benchmark constants. For $r_e = 1, r_r = 2$, $T_s = \delta = 0.002$, and $T_w = 0.004$, the source reports $u_* = 0.10953$, $N_W = 0.010039$, and thresholds $4\varepsilon_W(\alpha_{\min}, u_*)/\pi$ of $0.123, 0.0146, 0.0036$ at $\alpha_{\min} = 0.6, 0.7, 0.8$, respectively. For $T_s = \delta = 2 \times 10^{-4}$, it reports $u_* = 0.03464$ and thresholds $0.0123, 0.00145, 0.00036$. Unlike u_*, N_W depends on the full pulse and window shapes; its decimal value is not determined by the support lengths alone.

SUPPLIED waveform summaries. The original source gives backscatter front values $-0.08286, -0.28452, -0.79747, -1.73205$ at $\alpha = 0.95, 0.85, 0.7, 0.6$, with reported peak echoes $0.0030, 0.0114, 0.0372, 0.0815$ for its pulse. For $r_e = 1, r_r = 1.5, \theta = 0.9\pi\alpha$, it reports geometric arrivals $1.405, 1.639$ at $\alpha = 0.4$, and $1.219, 1.425, 2.497$ at $\alpha = 1/3$, with possible diffraction onset $L = 2.5$. The latter pure-image case has no diffracted wake. These summaries replace no raw waveform archive.

OPEN numerical certification package. A certified computational release would need the exact scripts and inputs, software versions and precision settings, raw outputs, truncation and quadrature bounds, mode-tail bounds, calibration enclosures, and a complete propagation of these errors to the reported functional and record law. None is inferred from the residual magnitudes above.

Release theorem and remaining obligations

Theorem (ANALYTIC: scoped mathematical release). *Under the explicitly stated scalar kernel input and the hypotheses of the relevant results, this revision releases:*

1. *the general twisted front formula , its scalar and backscatter specializations, and the additive high-frequency asymptotic ;*

2. *the exact averaging formula , the zero-deficit witness on* $1/2 < \alpha \le 1$*, and explicit window separation on compact sectors avoiding backscatter shadow values;*

3. *finite-window injectivity under , with image-isolated windows also available in compact sectors below* $1/2$*;*

4. *complete destructive clock/report and signal channels, their surviving residuals, and the instrumented total-variation estimate under the supplied preparation assumption;*

5. *twisted-scalar and spin-1 component front identities, with no automatic transfer to a gauge-invariant detector or a higher-spin core family;*

6. *neutral smooth-core selection, exact pre-core-window agreement, convergence of smeared records, and finite-precision indistinguishability;*

7. *the explicit resolvent-to-record estimate and the exterior matching coefficient . The numerical values in Section retain SUPPLIED provenance and UNCERTIFIED accuracy status. A physical acquisition channel, an explicit profile-uniform core decay rate, global fixed-resolution separation through shadow values, and the specified twisted/higher-spin core extensions remain OPEN.*

Proof. Items (i)–(iii) are Lemma , Theorems , , , and Lemmas , . Item (iv) is Theorems , , , and Proposition . Item (v) is Propositions and , with their stated interpretation. Item (vi) is Lemma , Theorem , and Corollary . Item (vii) is Theorem and Proposition . The remaining status statements identify assumptions, missing estimates, and documentary reports rather than adding mathematical conclusions.

par *Non-extraction and non-sourcing boundary.* The conclusion concerns a continuum functional and, only under the declared preparation assumption, a finite record law. Destructive Enter does not acquire that functional, select the host, or turn a conserved tensor into a source of the cone. No geometry update or Einstein equation is inferred [2, 9, 7, 10].

What has changed from the supplied first version

The principal corrections are mathematical rather than cosmetic:

1. A vanishing leading front coefficient is handled by an additive Fourier asymptotic. No relative error is formed by division by zero.

2. The front formula excludes shadow values and allows removable singularities. The total kernel, not its separated

singular pieces, governs a shadow-line calculation.

3. A positive calibration N_W is required. An arbitrary nonnegative window does not automatically define a usable ratio.

4. Exact continuum integrals are distinguished from finite report codes and stochastic signal outcomes. The signal instrument does not output exact real amplitudes.

5. The image-window problem is addressed on compact non-shadow sectors by isolating the earliest direct pulse and ending all image pulses before the echo gate. A global fixed-resolution theorem is not claimed.

6. Smoothing robustness retains the no-leakage hypotheses. Unrestricted smoothing can invalidate the witness.

7. Core convergence is not called exact metric non-identifiability. The proved conclusion is finite-precision indistinguishability.

8. The post-core quantitative question is reduced to a precisely normalized exterior matching estimate. The remaining uniform decay rate is not hidden behind a strong-resolvent theorem.

9. Spin-1 component identities are not promoted to unrestricted gauge-invariant polarimetry. Twisted and spin-2 core families are not invented by relabeling scalar channels.

10. Unrun numerical reports and their nonzero residuals remain visible, including the reported maximum ratio 0.252.

Successor obligations

A successor wishing to make a stronger operational or physical claim must supply the corresponding missing object:

1. a field representation and physical continuum-to-register acquisition channel, with a complete comparison including failures and residuals;

2. resource and causal-embedding certificates for the declared echo route, beyond the finite CPTP assembly

proved here;

3. a mode/frequency-uniform estimate on sufficient for a stated decay rate in ;

4. treatment of total-kernel shadow transitions and a specified arrival-resolution model for any global separation claim;

5. a spatially smeared separation theorem if extended emitters or receivers are used in the deficit experiment;

6. the connection, operators, boundary domains, compatibility identities, and convergence theorem for each proposed twisted or higher-spin core family;

7. exact computational artifacts and total numerical error enclosures before any supplied decimal comparison is certified.

Standing non-claims. No metric reconstruction, Einstein dynamics, backreaction, observational cosmic-string prediction, interacting field product, quantum Ward identity, characteristic graph admission, or spacetime emergence is constructed. No direction is certified as physical. The cone, deficit, extension, and acquisition interface remain inputs. The operator-versus-contact distinction is unchanged. OpenAI catalogue entries are external records, not premises.

$$\text{boxed}\qquad \begin{array}{c}\text{continuum echo functional + supplied finite preparation + complete finite instrument}\\ \Rightarrow \text{finite record law and correct declared residual,}\\ \text{not continuum acquisition, metric reconstruction, or sourcing,}\\ \mathrm{O.1} = \mathrm{texttt}\,\mathit{NOT_CLOSED}.\end{array}$$

appendix

A minimal task and error declaration

For reproducibility, a concrete echo task should state at least:

1. the host identifier, α-sector, bundle twist, extension, source and receiver positions or spatial profiles;

2. the exact pulse, windows, support margins, clock labels, and a positive lower enclosure for N_w;

3. whether the requested real task is y_d, y_e, their ratio, the calibrated ratio, or a real/imaginary phase component;

4. finite output ranges, code precision, rounding rules, failure and unused words, and every supplied numerical enclosure;

5. the signal parameters r, v, σ, preparation-value error, residual preparation error, and complete composition error;

6. every surviving finite register and untouched-reference comparison, together with consumed-name declarations;

7. which entries are analytic, supplied, uncertified, or open. A supplied task sheet is not a proof of its acquisition assumptions.

Source and verification declarations

The supplied document is the retained source for the legacy numerical reports and for version-qualified companion descriptions not independently audited in full. Bibliographic references identify the relevant articles and manuscript records; they do not imply that every cited companion proof was reconstructed or machine-checked here.

This source is designed as one pdfLaTeX file with an internal bibliography and no external graphics. No successful compilation, independent numerical execution, proof-assistant verification, independent referee approval, repository installation, experiment, or journal acceptance is asserted.

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