I-B-G-Q-B: one interacting product, still not an admission

A separation sheet: a constructed product with its Ward identity, and a scoped obstruction for the inherited coefficient

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Abstract

This new sheet takes the compatibility criterion, the finite part and the three version-5 typing propositions of I-B-G-Q as inputs and proves two things. First, for one declared interaction (a massless complex scalar coupled to an external $U(1)$ field on auxiliary Euclidean $\mathbb{R}^4$) we construct the renormalized vacuum-polarization product $\Pi^{(v)} = c_0(\partial\partial - \delta\Delta)u_v$, $c_0 = -1/(48\pi^4)$. We prove that it extends the off-diagonal product, satisfies the interacting Ward identity exactly, is covariant, and is unique up to the scale v under the stated degree bound. For it, descent, Ward identity and normalization hold together, while scale independence fails. We prove that it is not the inherited coefficient: its singular support is a point, whereas the inherited datum is singular along a codimension-two set. Second, under the stated typing we prove that the inherited coefficient is not determined by the inputs: an injective, identically conserved worldsheet family $\Theta(\mathscr{D}'(W))$ preserves every typed datum, and the contractible incidence closure supplies no monodromy constraint. We also prove that the typing alone decides neither success nor failure of the actual-image condition, and we record a conditional support separator. Its hypothesis, a nonzero defect of the actual lift, is not established. No quantum coefficient is admitted, and the certificate stays RG-open.

Version 2 adds: the exact running of $\Pi^{(v)}$ in position and momentum space, $v\partial_v\Pi^{(v)} = -\frac{1}{24\pi^2}(\partial\partial - \delta\Delta)\delta_0$ and $\widehat{\Pi}^{(v)} = \frac{1}{48\pi^2}(\delta k^2 - kk)(\log(k^2/v^2) - c_\star/\pi^2)$ (Theorem); a renormalized product on regular balls of the Euclidean section of the prescribed cone for deficits $\alpha = 1/N$, by images, with exact Ward identity, a seagull term forced by

the Ward identity, N-independent running, and coincidence value $(\alpha^{-2} - 1)/(48\pi^2 r^2)$ (Theorem); the precise status of the Lorentzian and general-deficit transfer (Theorem); and the exact dimension of the translation-invariant free family, $25, 45, 71$ for $\omega = 4, 5, 6$ ($3, 3, 5$ with covariance), which equals the number of real normalization data that are necessary and sufficient (Theorem).

Version 3 corrects the v2 singularity hypothesis (T0), which the benchmark $|x_\perp|^{-2}$ does not satisfy, to a non-removable-smooth-singularity hypothesis under which the non-admission theorem holds. It proves that the Euclidean product and all its proved properties do not determine a flat Lorentzian product, even modulo local counterterms (a two-parameter family with identical Ward identity, scaling and running and non-local differences), and writes one explicit declared Lorentzian datum; its identification with the time-ordered product is not proved. It proves that finite images realize the cone exactly for $\alpha = 1/N$, that the typed inputs determine neither the value nor the sign of the actual-lift detecting functional, and that in every degree all $3m^2 + 5m + 3$ normalization slots are left unfilled by every subset of the typed inputs. No quantum coefficient is admitted; the certificate remains RG-open. Version 4 identifies the declared Feynman datum as i times the unitary Epstein–Glaser time-ordered product, unique up to the scale, with unitarity as exactly the extra datum; constructs the Euclidean cone product for every deficit by a closed-form mode sum; and evaluates the defect of a naive lift on the declared model (local, of size $(\alpha^{-2} - 1)/(24\pi^2 r^2)$, removable). Version 5 constructs the Lorentzian local cone remainder for every deficit by analytic continuation of the closed form (real, symmetric, a bisolution, equal to the image sum at $\alpha = 1/N$) and the Ward-fixed seagull $2\mathrm{i}H^L_\alpha(x, x)$, without a state or time-ordered identification; evaluates the defect for a whole class of lifts; shows that the running does not place the product outside the Oai282-A Ward certificate; and counts the normalization slots. Version 6 determines the global singular structure of the continued Lorentzian remainder (image-cone poles with residue $1/\sinh u_*$, a diffracted pole whose residue vanishes iff $\alpha = 1/N$, conjugate branches beyond the diffracted front) and turns it into a method obstruction: the seagull $s = 2\mathrm{i}H$ is global on the diagonal, but globalizing off it requires a boundary-value prescription that the inputs do not supply. It shows that the defect class of every lift agreeing off the diagonal is zero, with

undetermined representative.

No quantum coefficient is admitted. No compilation is asserted. Cited sheets are documentary, never premises.

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Book expansion: separation table, running cross-cite, missing box

toprule Landmark Relation Separation

midrule I-B-G-Q Certificate criteria as inputs Criteria ≠ admission of inherited coefficient

I-B-G Scalar Green functions Green functions ≠ the product

Q-C / CERT Later Ward / slot statements Cross-citations under local hypotheses only

Oai282-A Ward certificate comparison Documentary separation

bottomrule

Exact running (book-visible restatement)

Theorem records

$$v\partial_v \Pi^{(v)} = -\frac{1}{24\pi^2}(\partial\partial - \delta\Delta)\delta_0$$

on the declared free-product model, together with the flat Euclidean momentum-space form. Proposition : finite, conserved, covariant, and normalized still runs. No RG-closure follows. This is a restatement under local hypotheses, not an admission of the inherited characteristic coefficient.

Still missing. Full characteristic product; output-space membership; interacting Ward; RG-closed certificate; Host C; quantum coefficient. Finite+conserved≠non-running remains in force.

Changes to v5

• **New results (v6).** Sections –: Theorem , Corollary , Definition , Theorem .

• **Targets.** (1) Globalizing Theorem : *method obstruction* (Theorem , Corollary); the singular set and residues are exact; the continuation is real for $\alpha = 1/N$ and not real in general beyond the diffracted front (numerics, C6-Q-5); a global object requires a boundary-value (Sokhotski–Plemelj) prescription, and for $\alpha \notin \{1/N\}$ the Carslaw diffracted term. (2) Actual-lift defect classes *tightened*: zero for every (OD) lift, representative undetermined, $\lfloor p/2 \rfloor$ slots under (LC) (Theorem). (3) RG-open stays.

• **Corrections to v5.** No v5 statement was found false. The v5 note that VI-DST thm:replacement returned 404 is

superseded: VI-DST now resolves and is cited documentarily (Section).

• **Re-verification of v5 by different methods.** See Section ; checks C6-Q-3, C6-Q-4.

• **Unchanged.** No quantum coefficient is admitted; no admission; no bulk BV; no O.1; the product runs (RG-open).

Changes from v4 to v5 (retained record)

• **New results (v5).** Sections –: Lemma , Definition , Theorem , Definition , Theorem , Theorem , Definition , Theorems , , Proposition .

• **Plan items.** (1) Lorentzian general α: *constructed locally* for every $\alpha > 0$, with Ward-fixed seagull (Theorem); termwise summation is a method obstruction (Lemma); global product, state and TO identification *not* obtained (unitarity not supplied). (2) Defect *evaluated* for a class of lifts (Theorem); actual inherited lift not constructed. (3) Running vs Oai282-A: the plan claim is false; the certificate holds in formal transcription (Theorem). (4) Slots *counted* (Theorems ,). (5) Scale-independent Green functions are not the product (Proposition).

• **Corrections to v4.** No v4 statement was found false. The v4 open item ``Lorentzian cone product, $\alpha \notin \{1/N\}$, not constructed" is reduced: the local remainder and the Ward-fixed product on regular balls now exist; only the state/TO identification and the global object remain open.

• **Re-verification of v4 by different methods.** See Section ; checks RV5-Q-1 to RV5-Q-3.

• **Unchanged.** No quantum coefficient is admitted; no admission; no bulk BV; no O.1; scale independence fails (the product runs).

Changes from v3 to v4 (retained record)

• **New results (v4).** Sections –: Lemma , Theorem , Definition , Lemma , Theorem , Proposition .

• **Plan items.** (1) Feynman datum = i × unitary EG product; extra datum: unitarity (Theorem). (2) Beyond $\alpha = 1/N$: Euclidean product for every α by a different construction (Theorem); Lorentzian open. (3) Defect on a declared model: sign and size (Proposition).

• **Corrections to v3.** The non-claim of Proposition omitted a factor i: the datum restricts to i times the time-ordered product. The non-claim of Lemma (general-α value ``not used or claimed'') is superseded by Lemma .

• **Re-verification.** d_m by exact rank to $m = 9$ and generating function; i/(24π) by Plemelj (Section); checks C4-Q-1 to C4-Q-10, RV4-Q-1.

• **Unchanged.** No quantum coefficient is admitted; no admission; the product runs (RG-open); no bulk BV restoration.

Changes from v2 to v3 (retained record)

• **New results (v3).** Sections –: Definition , Lemma , Theorem , Proposition , Theorem , Proposition , Theorem , Corollary .

• **Plan items.** (1) Flat Lorentzian product: closed by the non-determination branch (Theorem); one declared datum is written (Proposition), but Theorem (a) stays conditional on its identification with the time-ordered product. (2) General α: method obstruction proved (Theorem). (3) Actual-lift defect: proved not determined (value or sign) by the typed inputs; not evaluated; no impossibility drawn (Proposition). (4) Normalization slots: gap certificate in every degree (Theorem , Corollary).

• **Corrections to v2** (Appendix): hypothesis (T0) and the proof of Theorem ; the ``applies verbatim'' remark after Theorem .

• **Re-verification.** Theorem by a hand proof in all degrees and floating-point SVD (C-Q-18, C-Q-19). Confirmed.

• **Unchanged.** Coefficient not admitted; RG-open; no bulk BV restoration; O.1 NOT_CLOSED.

Changes from v1 to v2 (retained record)

• **New results.** Sections –: Theorems , , , ; Lemma ; Propositions , ; Definition .

• **Withdrawn v1 statement.** v1 plan correction 2 said that a conical product ``is not claimed'' because Hadamard and wavefront control on the cone were not supplied. For $\alpha = 1/N$ that control is supplied here by images (Lemma

); the plan correction is replaced accordingly.

• **Corrected v1 remark.** The sentence after Theorem (b) (``with invariance, one real number remains free'') is the scalar count of I-B-G-Q; for the tensor class the free family has dimension 25 at $\omega = 4$ (Theorem). Theorem itself is unchanged and correct.

• **Re-verification.** Section ; checks C-Q-7 to C-Q-17.

• **Unchanged.** The flat product and its uniqueness, Theorem , the obstruction under the typing, Theorem , the conditional separator, ``RG-open''. Errors: Appendix .

Ledger

This is version 6 of a new sheet (changes: Section). It revises neither I-B-G-Q nor any finite-part or kernel sheet. Every cited sheet is documentary. The renormalized product of Section is defined and proved from first principles; the finite part it uses is re-proved (Lemma).

toprule Item Status Where

midrule endhead One renormalized interacting product **Constructed** Definition , Theorem .

Interacting Ward identity for that product **Recorded** (proved) Theorem (b).

Descent, Ward, normalization together **Hold** Theorem (i).

Scale independence for that product **Fails** (proved) Theorem (ii).

Identity with the inherited coefficient **Refused** (proved) Theorem .

Inherited coefficient under the stated typing **Obstruction proved** (non-determination) Theorem .

Failure of the actual-image condition Not proved; conditional separator only Theorem , Proposition .

Exact running (v2) **Proved** Theorem : $-\frac{1}{24\pi^2}\log(v'/v)(\partial\partial - \delta\Delta)\delta_0$; momentum form.

Product on the prescribed cone (v2) **Constructed** for $\alpha = 1/N$, Euclidean section, regular balls Theorem .

Lorentzian transfer, general α (v2) Precise status: one flat datum missing ($\alpha = 1/N$); method obstruction otherwise Proposition , Theorem .

Dimension of the free family (v2) **Computed**: $25, 45, 71$ ($3, 3, 5$ covariant) Theorem .

Hypothesis (T0) (v3) **Corrected** to NSS; Theorem valid under it Section , Appendix .

Flat Lorentzian product (v3) **Not determined** by the Euclidean data (theorem); one explicit declared datum with Ward and running; identification with the time-ordered product not proved Theorem , Proposition .

General α (v3) **Method obstruction proved**: finite images iff $\alpha = 1/N$ Theorem .

Actual-lift defect (v3) **Not determined** by the typed inputs (value or sign); not evaluated Proposition .

Normalization slots, all degrees (v3) **Unfilled** by every subset of the typed inputs; $d_m = 3m^2 + 5m + 3$ Theorem , Corollary .

Declared Feynman datum vs time-ordered product (v4) **Identified**: $\mathfrak{P}_v^F = \mathrm{i}\, T_v^F$, unique unitary EG extension up to v; extra datum = unitarity Theorem .

General α, Euclidean (v4) **Constructed** for every $\alpha > 0$ (closed-form mode sum); Lorentzian open Theorem .

Defect on a declared model (v4) **Evaluated**: $(\alpha^{-2} - 1)/(24\pi^2 r^2)$, diagonal, removable Proposition .

Lorentzian cone, general α (v5) **Constructed locally** for every $\alpha > 0$ (analytic continuation); Ward-fixed seagull $2\mathrm{i}H_\alpha^L$; no TO identification Theorem .

Defect for a class of lifts (v5) **Evaluated**: $(2H + a)\partial\delta$, removable Theorem .

Running vs Oai282-A Ward certificate (v5) **Not excluded** (formal transcription holds); plan claim corrected Theorem .

Slots of the general-α product (v5) $\lfloor p/2 \rfloor$ under (LC); infinite without Theorems , .

Globalizing Q-lorentzalpha (v6) **Method obstruction**: image-cone and diffracted poles; global on the diagonal ($s = 2\mathrm{i}H$); off it a boundary-value prescription is needed Theorem , Corollary .

Actual-lift defect classes (v6) **Class zero** for every (OD) lift; representative undetermined; $\lfloor p/2 \rfloor$ slots under (LC) Theorem .

Certificate RG-open Section .

Bulk BV restoration Not claimed —

bottomrule Standing ledger: No quantum coefficient is admitted. Host C is not created. O.1 stays NOT_CLOSED. XVI-N is not discharged. XV-B is not opened. H2 and I3 stay unresolved. The last reading bars (quantized observable, no external clock) are untouched. No compilation is asserted.

Certificate criteria as inputs

We restate the criteria in the form used. None is a premise of a proof below.

The simultaneous compatibility criterion. I-B-G-Q thm:compatibility considers an affine family $F_0 + a$ of corrections and four desired conditions at a reference scale: (C1) agreement with the canonical object on its canonical domain (``descent", $a|_H = -F_0|_H$); (C2) vanishing Ward defect; (C3) a specified normalization; (C4) literal scale independence ($\beta = 0$). It proves that (C1)–(C4) hold simultaneously exactly when the defect lies in the image of the linear map $\mathscr{L}$, and that the solution set is then an affine space over $\ker \mathscr{L}$.

Finite part and running. I-B-G-Q thm:finitepart constructs the subtracted finite part u_ν of $|x|^{-4}$ on the auxiliary Euclidean $\mathbb{R}^4$ and proves $\partial_{\log \nu} u_\nu = 2\pi^2 \delta_0$; I-B-G-Q prop:conserved-running shows that an exactly conserved tensor built from u_ν still has nonzero scale derivative. We use it as a warning: finiteness and conservation do not give non-running.

The v5 typing. I-B-G-Q prop:v5-codim2: for a codimension-two W, two extensions of a datum u° with transverse scaling degree $\leq \omega'$ differ by $\sum_{|\beta| \leq \omega' - 2} a_\beta(y) \otimes \partial_\perp^\beta \delta(x_\perp)$ with $a_\beta \in \mathscr{D}'(W)$; without translation invariance along W no finite set of real numbers fixes the extension. I-B-G-Q prop:v5-ward: point-supported, symmetric, conserved, covariant tensor distributions are exactly $Q(\mathrm{Box})(\partial^a \partial^b - \eta^{ab} \mathrm{Box})\delta_0$, a free module of rank one with no parity-odd

term. I-B-G-Q prop:v5-global: the flat incidence closure $\overline{\mathcal{N}^{\circ}}$ is contractible.

The release. I-B-G-Q thm:release(vi): no full characteristic map, no normalized inherited quantum coefficient, and no interacting Ward identity for that coefficient are constructed; the certificate is RG-open.

Finite parts and the kernel counterterm: named, not used

The plan names an I-B-VP finite-part label. The main I-B-VP sheet read for this run has no finite-part label; the finite-part section of the I-B-VP series is I-B-VP-1 sec:finiteparts, with the raw finite part I-B-VP-1 eq:rawfinitepart (plan correction, Appendix A). The XVI-O kernel counterterm is the contact correction of XVI-O prop:contacts, built on the analytic finite part XVI-O lem:analytic_FP_u_mu_R_o70. Both are recorded as documentary neighbours only: the product below uses neither, and it is not inserted into, or compared with, the XVI-O kernel.

One renormalized interacting product

Setting and the off-diagonal product

We work on the auxiliary Euclidean $\mathbb{R}^4$ of I-B-G-Q thm:finitepart, with $|x|$ the Euclidean norm, $\Delta = \sum_\mu \partial_\mu^2$, and $\delta_{\mu\nu}$ the Euclidean metric. Let φ be a massless complex scalar with two-point function

$$\langle \varphi(x)\varphi^*(y)\rangle = \Delta_0(x-y), \qquad \Delta_0(x) = \frac{1}{4\pi^2|x|^2}, \qquad -\Delta\Delta_0 = \delta_0,$$

$\langle \varphi\varphi\rangle = \langle \varphi^*\varphi^*\rangle = 0$, and Gaussian (Wick) higher correlations. Declare the interaction $V = e\int A^\mu j_\mu + e^2\int A_\mu A^\mu : |\varphi|^2 :$ with the current

$$j_\mu = : i(\varphi^*\partial_\mu\varphi - \partial_\mu\varphi^*\varphi) : ,$$

Wick ordered. The second-order matter product relevant to the two-point function of the external field A is the connected correlation $\langle j_\mu(x) j_\nu(0)\rangle$; the seagull term contributes $\langle : |\varphi|^2 : \rangle = 0$.

Lemma (Off-diagonal product). *On* $\mathbb{R}^4 \setminus \{0\}$,

$$\Pi^{\circ}_{\mu\nu}(x) := \langle j_\mu(x) j_\nu(0)\rangle = 2\,\partial_\mu\Delta_0\,\partial_\nu\Delta_0 - 2\,\Delta_0\,\partial_\mu\partial_\nu\Delta_0 = c_0\,(\partial_\mu\partial_\nu - \delta_{\mu\nu}\Delta)|x|^{-4},$$

with $c_0 = -1/(48\pi^4)$, *and* $(\partial_\mu\partial_\nu - \delta_{\mu\nu}\Delta)|x|^{-4} = -12|x|^{-6}(\delta_{\mu\nu} - 2x_\mu x_\nu/|x|^2)$.

Proof. Write $j_\mu(x) = i(\varphi^*_x\partial_\mu\varphi_x - \partial_\mu\varphi^*_x\varphi_x)$ and similarly at 0 with derivatives ∂'_ν in the second argument. In each of the four products only the pairings $\langle\varphi_x\varphi^*_0\rangle$ and $\langle\varphi^*_x\varphi_0\rangle$ are nonzero, each equal to $\Delta_0(x)$; a derivative in the second argument of $\Delta_0(x-y)$ at $y=0$ equals $-\partial_\nu\Delta_0(x)$. With the signs $(+,-,-,+)$ of the four products included, the four connected terms are

$$-\partial_\mu\Delta_0\partial_\nu\Delta_0, \qquad +\Delta_0\partial_\mu\partial_\nu\Delta_0, \qquad +\Delta_0\partial_\mu\partial_\nu\Delta_0, \qquad -\partial_\mu\Delta_0\partial_\nu\Delta_0,$$

and multiplication by $i^2 = -1$ gives the first equality of . Inserting $\Delta_0 = a|x|^{-2}$, $a = 1/(4\pi^2)$: $\partial_\mu\Delta_0 = -2ax_\mu|x|^{-4}$ and $\partial_\mu\partial_\nu\Delta_0 = a(-2\delta_{\mu\nu}|x|^{-4} + 8x_\mu x_\nu|x|^{-6})$, so the left side is $4a^2|x|^{-6}(\delta_{\mu\nu} - 2x_\mu x_\nu|x|^{-2})$. The stated derivative identity (check C-Q-2) then gives $c_0 = -4a^2/12 = -1/(48\pi^4)$; the whole identity is check C-Q-1.

par *Non-claim.* The lemma evaluates a free Wick contraction off the diagonal. It is not an interacting Ward identity, not a renormalized product, and not related to the inherited characteristic graph. par

The finite part

Lemma (Finite part, re-proved). *For* $v > 0$ *and* $\varphi \in C^\infty_c(\mathbb{R}^4)$ *the limit*

$$u_v(\varphi) = \lim_{\varepsilon \text{downarrow } 0} \Big(\int_{|x|>\varepsilon} \frac{\varphi(x)}{|x|^4}\,\mathrm{d}^4x + 2\pi^2\log(v\varepsilon)\varphi(0)\Big)$$

exists and defines $u_v \in \mathscr{D}'(\mathbb{R}^4)$ *extending* $|x|^{-4}$ *from* $\mathbb{R}^4\setminus\{0\}$. *It is* $O(4)$*-invariant,* $u_{v'} - u_v = 2\pi^2\log(v'/v)\delta_0$, *and with* $\varphi_\lambda(x) = \varphi(x/\lambda)$,

$$u_v(\varphi_\lambda) = u_{\lambda v}(\varphi) \qquad (\lambda > 0).$$

Proof. Since $\int_{\varepsilon<|x|<1}|x|^{-4} = 2\pi^2\log(1/\varepsilon)$, the bracket equals $\int_{\varepsilon<|x|<1}(\varphi-\varphi(0))|x|^{-4} + \int_{|x|\ge 1}\varphi|x|^{-4} + 2\pi^2\log v\,\varphi(0)$, and the first integrand is $O(|x|^{-3})$, integrable; continuity in φ follows from $|\varphi(x)-\varphi(0)| \le |x|\sup|\nabla\varphi|$. Away from 0 it is the integral against $|x|^{-4}$. Invariance and the v-relation are immediate. For substitute $x = \lambda y$:

$\int_{|x|>\varepsilon} \varphi(x/\lambda)|x|^{-4}\,\mathrm{d}^4x = \int_{|y|>\varepsilon/\lambda} \varphi(y)|y|^{-4}\,\mathrm{d}^4y$, and $\log(v\varepsilon) = \log(\lambda v \cdot \varepsilon/\lambda)$; let $\varepsilon/\lambda \to 0$. (Check C-Q-5 corroborates numerically.) This is the content of I-B-G-Q thm:finitepart, re-proved so as not to use it as a premise.

par *Non-claim.* The finite part is one subtraction scheme. Its scale v is a free datum; nothing here selects it. par

The product

Definition (One renormalized interacting product). For $v > 0$ set

$$\Pi^{(v)}_{\mu\nu'} = c_0\,(\partial_\mu\partial_{\nu'} - \delta_{\mu\nu'}\Delta)\,u_v \in \mathscr{D}'(\mathbb{R}^4; S^2\mathbb{R}^4),$$

(distributional derivatives). We call $\Pi^{(v)}$ the renormalized vacuum-polarization product of the declared interaction at scale v.

(We write ν' for the second tensor index only in this definition to avoid a clash with the scale; below the indices are μ, κ.)

Theorem (The product and its Ward identity).

1. *(Descent / extension.)* $\Pi^{(v)}|_{\mathbb{R}^4\setminus\{0\}} = \Pi^\circ$.

2. *(Ward identity.)* $\partial^\mu\Pi^{(v)}_{\mu\kappa} = 0$ *in* $\mathscr{D}'(\mathbb{R}^4)$*, for every* v.

3. *(Covariance.)* $\Pi^{(v)}$ *is symmetric and* $O(4)$*-covariant; its singular support is* $\{0\}$.

4. *(Uniqueness up to scale.) Every* $\Pi \in \mathscr{D}'(\mathbb{R}^4; S^2\mathbb{R}^4)$ *that extends* Π°*, is symmetric and* $O(4)$*-covariant, satisfies* $\partial^\mu\Pi_{\mu\kappa} = 0$*, and has scaling degree* $\mathrm{sd}_0(\Pi) \le 6$ *at* 0*, equals* $\Pi^{(v)}$ *for exactly one* $v > 0$.

Proof. (a) Off 0, $u_v = |x|^{-4}$ and derivatives commute with restriction; apply Lemma .

(b) $\partial^\mu(\partial_\mu\partial_\kappa - \delta_{\mu\kappa}\Delta) = \Delta\partial_\kappa - \partial_\kappa\Delta = 0$ on distributions (check C-Q-4 records the symbol identity).

(c) Symmetry is evident. For $R \in O(4)$, $u_v \circ R = u_v$ and the operator $\partial_\mu\partial_\kappa - \delta_{\mu\kappa}\Delta$ transforms as a symmetric 2-tensor. Off 0, Π° is smooth; at 0 it is not, being homogeneous of degree -6 and nonzero.

(d) Existence: $\Pi^{(v)}$ has the properties by (a)–(c); its scaling degree is 6, because gives $u_v(\lambda\,\cdot) = \lambda^{-4}(u_v + 2\pi^2 \log\lambda\,\delta_0)$ and hence $\Pi^{(v)}(\lambda\,\cdot) = \lambda^{-6}(\Pi^{(v)} + 2\pi^2 c_0 \log\lambda\,(\partial\partial - \delta\Delta)\delta_0)$, and $\lambda^{\omega}\cdot\lambda^{-6}\log\lambda \to 0$ for every $\omega > 6$. Uniqueness: let Π be another such extension. Then $D = \Pi - \Pi^{(v)}$ is supported in $\{0\}$, hence (Schwartz) $D = \sum_{|\alpha|\le k} c_\alpha \partial^\alpha \delta_0$ with constant symmetric-tensor coefficients. The terms of order $m = |\alpha|$ scale as λ^{-4-m}, so $\mathrm{sd}_0(D) \le 6$ forces $m \le 2$. $O(4)$ -covariance and symmetry leave exactly $\alpha\delta_{\mu\kappa}\delta_0 + \beta\partial_\mu\partial_\kappa\delta_0 + \gamma\delta_{\mu\kappa}\Delta\delta_0$: there is no first-order term, since a symmetric 2-tensor linear in one vector is not $O(4)$-covariant (it would be odd under $-I$), and ε-terms need four indices. The Ward identity for D holds iff $\alpha = 0$ and $\beta = -\gamma$ (check C-Q-3). Hence $D = -\gamma(\partial\partial - \delta\Delta)\delta_0$ for some $\gamma \in \mathbb{R}$. Since $\Pi^{(v')} - \Pi^{(v)} = 2\pi^2 c_0 \log(v'/v)(\partial\partial - \delta\Delta)\delta_0$ and $c_0 \ne 0$, the equation $2\pi^2 c_0 \log(v'/v) = -\gamma$ has exactly one solution $v' > 0$, and $\Pi = \Pi^{(v')}$. Distinct v' give distinct Π because $(\partial\partial - \delta\Delta)\delta_0 \ne 0$ (its trace is $-3\Delta\delta_0 \ne 0$, check C-Q-6).

par *Non-claim.* The Ward identity is proved for this one product of a free charged scalar on an auxiliary Euclidean chart. It is not the interacting Ward identity of the inherited coefficient, and it does not show that this product is that coefficient. par

The four conditions of the compatibility criterion

Define the normalization functional $\mathcal{N}(\Pi) = \langle \Pi_{\mu\mu}, \psi\rangle$ for a fixed radial $\psi \in C_c^\infty(\mathbb{R}^4)$ with $\Delta\psi(0) \ne 0$, and a prescribed value $C \in \mathbb{R}$.

Theorem (Three conditions hold together; the fourth fails). *For the family $\{\Pi^{(v)}\}_{v>0}$ of Theorem :*

1. *For every $C \in \mathbb{R}$ there is exactly one $v_C > 0$ such that $\Pi^{(v_C)}$ satisfies, simultaneously, descent (agreement with Π° off the diagonal), the Ward identity, and the normalization $\mathcal{N}(\Pi^{(v_C)}) = C$.*

2. *No member of the class described in Theorem (d) is scale independent: $\partial_{\log v}\Pi^{(v)} = 2\pi^2 c_0(\partial\partial - \delta\Delta)\delta_0 \ne 0$, and no $\Pi^{(v)}$ is homogeneous under dilations. No scale-independent finite counterterm can change the scale derivative. Hence, for this product, conditions (C1)–(C3) of I-B-G-Q thm:compatibility can be met simultaneously, while (C4) cannot: the actual-image condition of that theorem fails for this product.*

Proof. (i) $\mathcal{M}(\Pi^{(v)}) = \mathcal{M}(\Pi^{(1)}) + 2\pi^2 c_0 \log v \,\langle -3\Delta\delta_0, \psi\rangle = \mathcal{M}(\Pi^{(1)}) - 6\pi^2 c_0 \log v\, \Delta\psi(0)$, an affine bijection of $\log v \in \mathbb{R}$ onto $\mathbb{R}$ because $c_0 \Delta\psi(0) \neq 0$. Descent and Ward hold for every v. (ii) The derivative formula follows from Lemma ; it is nonzero by check C-Q-6. A scale-independent counterterm is annihilated by $\partial_{\log v}$. Non-homogeneity: the dilation formula in the proof of Theorem (d) has a nonzero $\log\lambda$ term. In the language of the criterion, the requirement $\beta = 0$ is not met by any member, so the defect is not in the image of $\mathscr{L}$.

par *Non-claim.* This theorem refutes non-running for this product; it says nothing about the scale behaviour of the inherited coefficient, which is not constructed. The certificate remains RG-open. par

The product is not the inherited coefficient

The admission problem typed in I-B-G-Q prop:v5-codim2 concerns extensions across a codimension-two set W of a datum u° given off W. We make explicit the hypothesis that the datum is genuinely singular along W:

1. $u^\circ \in \mathscr{D}'(\mathbb{R}^d \setminus W)$, $d = 4$, has a *non-removable smooth singularity* at every point of W (NSS): for no $w \in W$ is there a neighbourhood $V \ni w$ and $f \in C^\infty(V)$ with $f|_{V\setminus W} = u^\circ|_{V\setminus W}$. The benchmark $u^\circ = |x_\perp|^{-2} \otimes 1$ of I-B-G-Q prop:v5-codim2(iii) satisfies (T0): it is unbounded as $x_\perp \to 0$, so it has no continuous, let alone smooth, local extension (C-Q-23). *(Corrected in v3.* v2 stated (T0) as ``every point of W lies in the closure of sing supp u°''. The benchmark is smooth on $\mathbb{R}^4 \setminus W$, so its singular support there is empty and the v2 hypothesis *fails* for it; see Appendix .)

Theorem (Non-admission of the product). *Under (T0), no extension U of u° across W equals $\Phi^*\Pi^{(v)}$ (componentwise pull-back) for any diffeomorphism Φ between open subsets of $\mathbb{R}^4$ and any v. In particular the product of Definition is not the inherited characteristic coefficient of I-B-G-Q thm:release(vi) under the stated typing.*

Proof. Let U be an extension of u° on a domain meeting W, and $w \in W$ in the domain. If U were smooth near w, its restriction to a neighbourhood V would be a smooth f with $f|_{V\setminus W} = u^\circ$, contradicting (T0). So $W \cap \mathrm{dom} \subseteq \mathrm{sing\,supp}\, U$; W has dimension 2. The singular support of $\Phi^*\Pi^{(v)}$ is $\Phi^{-1}(\{0\})$, at most one point, by

Theorem (c), since diffeomorphisms preserve smoothness. A set containing an open piece of a 2-dimensional submanifold is not contained in a single point.

par *Non-claim.* This is a typing statement: the product and the inherited coefficient live in different admission problems. It does not assert that the inherited coefficient does not exist, and it does not use the inherited coefficient. par

Obstruction under the stated typing

We now prove that the current inputs cannot admit the inherited coefficient, by the non-determination pattern. Work on $\mathbb{R}^{1,3}$ with $W = \{x_\perp = 0\}$, $x_\perp = (x^2, x^3)$, $y = (x^0, x^1) \in W$, $\eta = \mathrm{diag}(-1, 1, 1, 1)$, $\mathrm{Box} = \eta^{ab}\partial_a\partial_b$. For a symmetric tensor datum u° conserved off W and $\omega \geq 0$ define the admissible class

$$\mathrm{Adm}(u^\circ, \omega) = \{U \in \mathscr{D}'(\mathbb{R}^4; S^2\mathbb{R}^4) : U|_{\mathbb{R}^4 \setminus W} = u^\circ,\ \mathrm{sd}_W(U) \leq \omega,\ \partial_a U^{ab} = 0\}.$$

The typing hypotheses are those recorded from v5:

1. codimension two: ambiguities are as in I-B-G-Q prop:v5-codim2(i);

2. free module: at a point, conserved covariant ambiguities are $Q(\mathrm{Box})(\partial\partial - \eta\mathrm{Box})\delta$ (I-B-G-Q prop:v5-ward);

3. contractible closure: $\overline{\mathscr{N}^\circ}$ is contractible (I-B-G-Q prop:v5-global), and locally path-connected and semilocally simply connected (true for the product model $\mathbb{R}^4 \times C$ with C a closed convex or polyhedral cone; we assume it).

Lemma (Conserved worldsheet family). *For $a \in \mathscr{D}'(W)$ put $\Theta(a)^{bc} = (\partial^b\partial^c - \eta^{bc}\mathrm{Box})(a \otimes \delta(x_\perp))$. Then $\Theta(a)$ is symmetric, supported in W, identically conserved, has $\mathrm{sd}_W(\Theta(a)) \leq 4$, and $a \mapsto \Theta(a)$ is linear and injective.*

Proof. Support and symmetry are clear; conservation is $\partial_b(\partial^b\partial^c - \eta^{bc}\mathrm{Box}) = 0$. Each term is a tangential derivative of a tensored with $\partial_\perp^\beta\delta(x_\perp)$, $|\beta| \leq 2$, whose transverse scaling degree is $2 + |\beta| \leq 4$ (as in I-B-G-Q prop:v5-codim2(i)). Injectivity: the trace is $\eta_{bc}\Theta^{bc} = (1 - 4)\mathrm{Box}\,(a \otimes \delta) = -3((\mathrm{Box}_W a) \otimes \delta + a \otimes \Delta_\perp\delta)$. If $\Theta(a) = 0$, pair the trace with $\varphi(y)\chi(x_\perp)$, χ even with $\chi = \frac{1}{4}|x_\perp|^2$ near 0: the δ term gives $\langle \mathrm{Box}_W a, \varphi\rangle\chi(0) = 0$ and the $\Delta_\perp\delta$ term

gives $\langle a, \varphi\rangle \Delta_{\perp}\chi(0) = \langle a, \varphi\rangle$. Hence $\langle a, \varphi\rangle = 0$ for all φ, so $a = 0$.

Theorem (The inherited coefficient is not determined by the typed inputs). *Assume (T0)–(T3) and* $\omega \geq 4$.

1. *If* $\mathrm{Adm}(u^{\circ}, \omega) \neq \varnothing$, *then for any U in it,* $U + \Theta(\mathscr{D}'(W)) \subseteq \mathrm{Adm}(u^{\circ}, \omega)$, *an infinite-dimensional affine family of distinct elements that agree on every typed input datum (the datum off W, the degree bound, conservation).*

2. *If, in addition, the data and the admitted extensions are invariant under translations along W, the family of (a) restricted to translation-invariant members still contains the one-parameter family* $U + t\,\Theta(1)$, $t \in \mathbb{R}$, *of distinct elements.*

3. *The global structure (T3) imposes no constraint that selects a member: every locally constant real normalization on* $\overline{\mathscr{N}^{\circ}}$ *is a single constant, every flat real line bundle (local system) on it is trivial, and so no monodromy or quantization condition on the normalization arises. Consequently the current inputs, under this typing, do not admit the inherited coefficient: admission requires a determined value, and none is determined.*

Proof. (a) For $U \in \mathrm{Adm}$ and $a \in \mathscr{D}'(W)$, $U + \Theta(a)$ agrees with U off W (Lemma , support), is conserved, and has $\mathrm{sd}_W \leq \max(\omega, 4) = \omega$. Distinctness and infinite dimension follow from injectivity of Θ and $\dim \mathscr{D}'(W) = \infty$. The typed inputs consist only of u°, the bound and conservation, all shared. (b) $\Theta(1)$ is translation invariant along W and nonzero by injectivity. This is consistent with I-B-G-Q prop:v5-codim2(ii): with invariance, one real number remains free. (c) Contractible spaces are path-connected, so a locally constant function is constant. With the local hypotheses in (T3), local systems of real lines correspond to homomorphisms $\pi_1 \to \mathbb{R}^{\times}$, and $\pi_1(\overline{\mathscr{N}^{\circ}}) = 0$; so every such local system is trivial and no holonomy constrains the constant. Neither statement fixes the value of the constant. The concluding sentence combines (a)–(c): every input datum is shared by an infinite (in the invariant case, at least one-parameter) family of admissible extensions.

par *Non-claim.* The obstruction is scoped to the stated typing and to the class Adm. It does not claim that no inherited coefficient exists in a larger class with additional normalization data, and it does not claim that the inherited coefficient differs from any particular candidate. This records the inherited coefficient as permanently

separated from the current inputs under that typing. No quantum coefficient is admitted. par

What the typing does not decide, and a conditional separator

Theorem is a *non-determination* statement. It is not a statement that the actual-image condition of I-B-G-Q thm:compatibility fails. We now show that the typing alone could not prove such a failure. We then isolate the one extra input that would.

Write the compatibility problem abstractly: $\mathscr{A}$ is a real vector space of corrections, $\mathscr{D}$ a defect space, $\mathscr{L} : \mathscr{A} \to \mathscr{D}$ linear, and $b \in \mathscr{D}$ the defect of a preliminary lift. All four conditions hold for the corrected candidate iff $b + \mathscr{L}a = 0$, i.e. iff $-b \in \operatorname{im} \mathscr{L}$, and the solutions are then $a_\star + \ker \mathscr{L}$ (this is the content of I-B-G-Q thm:compatibility).

Theorem (The typing alone decides neither success nor failure). *There are two compatibility problems with the same incidence space (contractible), the same free correction module of rank one, the same codimension-two extension interface, and the same defect b, such that $-b \in \operatorname{im} \mathscr{L}$ in the first and $-b \notin \operatorname{im} \mathscr{L}$ in the second.*

Proof. Take the incidence space $\mathbb{R}^4$, $\mathscr{A} = \mathbb{R}e$, $\mathscr{D} = \mathbb{R}^4$, $b = (0, 1, 0, 0)$, $\mathscr{L}_1(ae) = (0, a, 0, 0)$ and $\mathscr{L}_0 = 0$. Then $a = -1$ solves $b + \mathscr{L}_1 a = 0$, and $b + \mathscr{L}_0 a = b \neq 0$ for all a. The codimension-two interface (extension changes $a \otimes \partial_\perp^\beta \delta(x_\perp)$ across W) and (T1)–(T3) concern the space in which corrections live and the shape of their ambiguity. They do not specify the response map $\mathscr{L}$, so both problems carry the same typing.

par *Non-claim.* This is Theorem Q-typing-insufficient of the second user draft, re-proved. The two problems are abstract counterexamples to an inference, not realizations of the frozen graph. The theorem limits what a *failure* statement can rest on. It does not weaken Theorem , which asserts non-determination, not failure: in either problem the typed inputs (without a normalization datum) leave the extension undetermined. No quantum coefficient is admitted. par

Proposition (Conditional support separator). *In the abstract problem, let $\mathscr{D}$ consist of distributions on an open set $\mathscr{W}$, and let $\mathscr{L}$ be support-non-increasing (*$\operatorname{supp} \mathscr{L}a \subseteq \operatorname{supp} a$*; this holds for restriction, differential operators*

and the scale derivative). Suppose every $a \in \mathscr{A}$ is supported in a closed set $Z \subset \mathscr{W}$, and let $U \subset \mathscr{W} \setminus Z$ be open. If $b|_U \neq 0$, then $-b \notin \operatorname{im} \mathscr{L}$: no correction in $\mathscr{A}$ meets all four conditions. Equivalently, for any $\varphi \in C_c^\infty(U)$ with $\langle b, \varphi \rangle \neq 0$, the functional $\Lambda = \langle \cdot, \varphi \rangle$ vanishes on $\operatorname{im} \mathscr{L}$ and not on b.

Proof. $(\mathscr{L}a)|_U = 0$ for every a, since $\operatorname{supp} \mathscr{L}a \subseteq Z$. If $b + \mathscr{L}a = 0$, restricting to U gives $b|_U = 0$.

par *Non-claim.* The v5 typing places corrections on coincidence diagonals and on the worldsheet, so the proposition applies on any open set avoiding both. It yields a failure of the actual-image condition *only if* the defect b of the actual preliminary lift is nonzero there. That lift is not constructed in the supplied material (I-B-G-Q thm:release(vi)), and its defect is not evaluated here. The witnesses offered for this in the third user draft are a degree-filtered probe defect and an ambiguity element of XIV-K-ST, not the defect of the actual lift. So no failure is recorded, and the inherited coefficient is not proved impossible. No quantum coefficient is admitted. par

Running reminder

Proposition (Finite and conserved does not mean non-running). *The constructed product is finite, conserved, covariant, and normalized, and it runs: $\partial_{\log v} \Pi^{(v)} \neq 0$. Hence no RG-closure of the certificate follows from these properties.*

Proof. Theorems and (ii).

par *Non-claim.* This is the warning of I-B-G-Q prop:conserved-running, re-derived for a product of the declared interaction. The certificate remains RG-open. par

v2: the exact running of Π^{(v)}

Theorem (Exact running, position and momentum space). *For $v, v', \lambda > 0$:*

1. $\Pi^{(v')} - \Pi^{(v)} = -\frac{1}{24\pi^2} \log \frac{v'}{v} (\partial\partial - \delta\Delta)\delta_0$ *exactly; equivalently* $v\partial_v \Pi^{(v)} = -\frac{1}{24\pi^2}(\partial\partial - \delta\Delta)\delta_0$.

2. $\Pi^{(v)}(\lambda\, \cdot) = \lambda^{-6}(\Pi^{(v)} - \frac{1}{24\pi^2} \log \lambda\, (\partial\partial - \delta\Delta)\delta_0)$.

3. *There is a universal constant $c_\star$ such that $\widehat{u_v}(k) = -\pi^2 \log(k^2/v^2) + c_\star$ as tempered distributions, and*

$$\widehat{\Pi}^{(v)}_{\mu\kappa}(k) = \frac{1}{48\pi^2}(\delta_{\mu\kappa}k^2 - k_\mu k_\kappa)(\log\frac{k^2}{v^2} - \frac{c_\star}{\pi^2}).$$

The coefficient of $\log k^2$ is $1/(48\pi^2)$; with the normalization $\frac{1}{2}e^2$AΠA of the second-order effective action this is the familiar one-loop scalar-electrodynamics coefficient (documentary comparison only).

Proof. (a) Lemma and Definition give the difference $2\pi^2 c_0 \log(v'/v)(\partial\partial - \delta\Delta)\delta_0$, and $2\pi^2 c_0 = -1/(24\pi^2)$ (C-Q-12). (b) is the formula in the proof of Theorem (d), with the same constant; its sign is re-derived as follows: $\langle u_v(\lambda\,\cdot), \varphi\rangle = \lambda^{-4} u_v(\varphi(\cdot/\lambda)) = \lambda^{-4} u_{\lambda v}(\varphi)$ by , and $u_{\lambda v} = u_v + 2\pi^2 \log\lambda\,\delta_0$; two derivatives contribute λ^{-2}. (c) $\widehat{u_v}$ is a rotation-invariant tempered distribution, and the Fourier transform of the dilation relation reads $\widehat{u_v}(k/\lambda) = \widehat{u_v}(k) + 2\pi^2 \log\lambda$. Hence $v = \widehat{u_v} + \pi^2 \log k^2$ is rotation invariant and dilation invariant (homogeneous of degree 0). On $\mathbb{R}^4 \setminus 0$ such a distribution is a constant; degree $0 > -4$, so its extension across 0 is unique and v is constant on $\mathbb{R}^4$. Since $u_{v'} - u_v = 2\pi^2 \log(v'/v)\delta_0$ and $\widehat{\delta}_0 = 1$, the constant is $\pi^2 \log v^2 + c_\star$. Finally $\widehat{\partial_\mu\partial_\kappa - \delta_{\mu\kappa}\Delta} = \delta_{\mu\kappa}k^2 - k_\mu k_\kappa$ and $c_0 \cdot (-\pi^2) = 1/(48\pi^2)$ (C-Q-12). Independently, the analytic family $\widehat{|x|^{-4+2\varepsilon}} = \pi^2 4^\varepsilon \Gamma(\varepsilon)\Gamma(2-\varepsilon)^{-1}|k|^{-2\varepsilon}$ has $\log k^2$ coefficient $-\pi^2$ at order ε^0 (C-Q-12; the general formula is corroborated numerically in C-Q-13).

par *Non-claim.* This is the exact scale dependence of one product of a free charged scalar on the auxiliary chart. It is not the running of the inherited coefficient, which is not constructed, and the certificate remains RG-open. No quantum coefficient is admitted. par

v2: a product on the prescribed conical setting

v1 recorded (plan correction) that a product on the cone would need Hadamard and wavefront control not supplied there. For the deficits $\mu = 2\pi(1 - 1/N)$, $N \geq 2$, i.e. $\alpha = 1/N$, this control can be supplied by the method of images, and we construct the product on regular balls. For other α we record precisely what is missing.

Euclidean section and the image Green function

The Euclidean section ($T = -\mathrm{i}\tau/c$) of the prescribed static metric of I-B-G-Q sec:setting (I-B-G-Q prop:geometry) is $\mathrm{d}\tau^2 + \mathrm{d}r^2 + \alpha^2 r^2\,\mathrm{d}\varphi^2 + \mathrm{d}z^2$. With the developed angle $\psi = \alpha\varphi \in [0, 2\pi\alpha)$ and $\alpha = 1/N$, it is the orbifold $\mathbb{R}^4/\mathbb{Z}_N$, where R rotates the $(X, Y) = (r\cos\psi, r\sin\psi)$ plane by $2\pi/N$ and fixes (τ, z). Let $G_0(x) = 1/(4\pi^2|x|^2)$ and

$$G_N(x, x') = \sum_{k=0}^{N-1} G_0(x - R^k x'), \qquad H(x, x') = \sum_{k=1}^{N-1} G_0(x - R^k x').$$

Lemma (Regular balls). *Let p be a point at distance $r_p > 0$ from the axis, $s = \sin(\pi/N)$, and V the ball of radius $\rho < r_p s/(1+s)$ about p. Then V embeds isometrically in the cone; for $x, x' \in V$ and $k \neq 0$, $|x - R^k x'| \geq 2(r_p - \rho)s - 2\rho > 0$; H is smooth, real, symmetric ($H(x,x') = H(x',x)$) and harmonic in each variable on $V \times V$; $-\Delta_x G_N(\cdot, x') = \delta_{x'}$ on V; and*

$$H(x, x) = \frac{N^2 - 1}{48\pi^2 r(x)^2} = \frac{\alpha^{-2} - 1}{48\pi^2 r(x)^2}.$$

Proof. $|x' - R^k x'| = 2r(x')\sin(\pi k/N) \geq 2r(x')s$ (C-Q-8) and $r(x') \geq r_p - \rho$, so the triangle inequality gives the lower bound, which is positive by the choice of ρ; in particular the images $R^k V$ are pairwise disjoint and V embeds. Each $G_0(x - R^k x')$, $k \neq 0$, is then smooth on $V \times V$ and harmonic in x (C-Q-7). Symmetry: $G_0(x - R^k x') = G_0(R^{-k}x - x') = G_0(x' - R^{N-k}x)$ (C-Q-7), and $k \mapsto N - k$ permutes $\{1, \ldots, N-1\}$. $-\Delta G_0 = \delta_0$ and the image terms are harmonic on V. Coincidence: $|x - R^k x|^2 = 4r^2\sin^2(\pi k/N)$ and $\sum_{k=1}^{N-1} \csc^2(\pi k/N) = (N^2 - 1)/3$ (C-Q-8).

par *Non-claim.* The lemma uses only the declared Euclidean section and $\alpha = 1/N$. For a massless scalar on a straight cosmic string a closed form $(\alpha^{-2} - 1)/(48\pi^2 r^2)$ for general α is reported in the literature; it is mentioned only as a documentary comparison, and is not used or claimed here for $\alpha \notin \{1/N\}$. par

The product and its Ward identity

Wick contractions with $\langle \varphi(x)\varphi^*(x') \rangle = G_N(x, x')$ give off the diagonal (the computation of Lemma with G_N in place of $\Delta_0(x - y)$)

$$\Pi^{\circ}_{N,\mu\nu}(x,x') = -(2\partial_\mu G_N\,\partial'_\nu G_N - 2G_N\,\partial_\mu\partial'_\nu G_N), \qquad x \neq x',$$

with ∂' acting on x'. Splitting $G_N = G_0(x-x') + H$ gives $\Pi^{\circ}_N = \Pi^{\circ}(x-x') - X_{\mathrm{cr}} - X_{HH}$ with

$$X_{\mathrm{cr}} = 2\partial_\mu G_0\,\partial'_\nu H + 2\partial_\mu H\,\partial'_\nu G_0 - 2G_0\,\partial_\mu\partial'_\nu H - 2H\,\partial_\mu\partial'_\nu G_0, \qquad X_{HH} = 2\partial_\mu H\,\partial'_\nu H - 2H\,\partial_\mu\partial'_\nu H.$$

In X_{cr}, $G_0 = G_0(x-x')$ is locally integrable, its derivatives are taken in the sense of distributions, and each product is a smooth function times a distribution; no renormalization is involved.

Definition (Cone product). On $V \times V$ put

$$\Pi^{(v)}_{N,\mu\nu}(x,x') = \Pi^{(v)}_{\mu\nu}(x-x') - X_{\mathrm{cr},\mu\nu} - X_{HH,\mu\nu} - 2\,\delta_{\mu\nu}\,H(x,x)\,\delta(x-x').$$

The last term is the seagull contribution $\langle : |\varphi|^2 : \rangle$ of the declared interaction, with Wick ordering relative to the local flat parametrix G_0.

Theorem (Cone product: extension, Ward identity, running). *For $N \geq 2$, a regular ball V as in Lemma , and $v > 0$:*

1. *$\Pi^{(v)}_N$ restricts to Π°_N on $V \times V$ minus the diagonal.*

2. *(Ward identity) $\partial^{\mu}_x \Pi^{(v)}_{N,\mu\nu} = 0$ in $\mathscr{D}'(V \times V)$.*

3. *(The seagull is forced) Among the distributions $\Pi^{(v)}(x-x') - X_{\mathrm{cr}} - X_{HH} + a(x)\delta_{\mu\nu}\delta(x-x')$ with $a \in C^{\infty}(V)$, the Ward identity holds iff $a(x) = -2H(x,x)$.*

4. *(Running) $v\partial_v \Pi^{(v)}_N = -\frac{1}{24\pi^2}(\partial\partial - \delta\Delta)_x \delta(x-x')$, independent of N.*

5. *(Uniqueness under local covariance) If Π extends Π°_N, is symmetric, conserved in x, has scaling degree ≤ 6 at the diagonal, and $\Pi - \Pi^{(1)}_N = \sum_{|\gamma| \leq K} c_\gamma\,\partial^{\gamma}_x \delta(x-x')$ with constant tensor coefficients that are $O(4)$-covariant in the local flat chart (hypothesis (LC)), then $\Pi = \Pi^{(v)}_N$ for exactly one $v > 0$.*

Proof. (a) Off the diagonal $\Pi^{(v)}(x-x') = \Pi^{\circ}(x-x')$ (Theorem (a)), the distributional products agree with the

pointwise ones, and the seagull term vanishes. (b) $\partial^{\mu}\Pi^{(v)}(x-x') = 0$ by Theorem (b). By the Leibniz rule (C-Q-9), $\partial^{\mu}X_{\mathrm{cr}} = 2\Delta G_0\,\partial'_{v}H + 2\Delta H\,\partial'_{v}G_0 - 2G_0\,\partial'_{v}\Delta H - 2H\,\partial'_{v}\Delta G_0$. With $\Delta H = 0$, $\Delta G_0(x-x') = -\delta(x-x')$ and $\partial'_{v}\Delta G_0 = \partial_{v}\delta(x-x')$,

$$\partial^{\mu}X_{\mathrm{cr}} = -2(\partial'_{v}H)\delta - 2H\,\partial_{v}\delta = 2[(\partial_{v}H) - (\partial'_{v}H)](x',x')\,\delta(x-x') - 2H(x',x')\,\partial_{v}\delta(x-x'),$$

using $H(x,x')\partial_{v}\delta(x-x') = H(x',x')\partial_{v}\delta(x-x') - (\partial_{v}H)(x',x')\delta(x-x')$. By symmetry of H the bracket vanishes (C-Q-11). Also $\partial^{\mu}X_{HH} = 2\Delta H\,\partial'_{v}H - 2H\,\partial'_{v}\Delta H = 0$ (C-Q-10). Finally $\partial^{\mu}[\delta_{\mu\nu}a(x)\delta(x-x')] = \partial_{v}[a(x')\delta(x-x')] = a(x')\partial_{v}\delta(x-x')$. Hence $\partial^{\mu}\Pi = (2H(x',x') + a(x'))\partial_{v}\delta(x-x')$, which vanishes with $a = -2H(x,x)$. (c) The last display: the coefficient of $\partial_{v}\delta(x-x')$ vanishes iff $a = -2H(\cdot,\cdot)$, since $f(x')\partial_{v}\delta(x-x') = 0$ forces $f = 0$. (d) Only $\Pi^{(v)}(x-x')$ depends on v; apply Theorem (a). (e) $\Pi - \Pi_N^{(1)}$ is supported on the diagonal, has scaling degree ≤ 6 and is conserved; under (LC) the argument of Theorem (d) applies verbatim in the local chart and gives $-\gamma(\partial\partial - \delta\Delta)\delta$, and (d) gives exactly one v.

par *Non-claim.* This is one product of a free charged scalar on regular balls of the Euclidean section of the prescribed cone, for $\alpha = 1/N$ only. It is not the inherited coefficient: it is a bilocal distribution on $V \times V$, a different ambient type from the single-variable worldsheet datum of Theorem , so they are separated by type, and no identification between the two types is used. (v2 said Theorem ``applies verbatim"; it does not, since that theorem compares distributions on open subsets of $\mathbb{R}^4$; corrected in v3, Appendix .) It is not a Lorentzian product, and not an interacting Ward identity of the inherited theory. No quantum coefficient is admitted. par

Proposition (Without local covariance the cone product is not unique). *For every $f \in C^{\infty}(V)$, $\Pi_N^{(v)} + f(x')(\partial\partial - \delta\Delta)_x\delta(x-x')$ satisfies (a), (b) and the scaling-degree bound of Theorem , and these are pairwise distinct for distinct f.*

Proof. The added term is supported on the diagonal, has scaling degree 6, and $\partial_x^{\mu}(\partial_{\mu}\partial_{\nu} - \delta_{\mu\nu}\Delta)_x = 0$ while $f(x')$ is constant in x. Distinctness: pair with $\varphi(x)\psi(x')$ and take the trace, $-3f(x')\Delta_x\delta$.

par *Non-claim.* Hypothesis (LC) is therefore a genuine extra datum on the cone; it is not supplied by the inputs. The proposition does not assert that physically acceptable products violate (LC). par

Lorentzian transfer and general deficits: what is and is not supplied

Proposition (Lorentzian image state for $\alpha = 1/N$). *On the prescribed Lorentzian cone with $\alpha = 1/N$, let W_0 be the Minkowski vacuum two-point function of the massless scalar and $W_N(x,x') = \sum_{k=0}^{N-1} W_0(x, R^k x')$. Then W_N is the two-point function of a quasi-free state of the field on the cone: it is positive, solves the wave equation in each variable, and is R-invariant. On a ball V about a regular point of radius $\rho < r_p s/(2+s)$ (in an inertial chart of the developed space), $W_N - W_0$ is smooth, real and symmetric on $V \times V$, with the same coincidence value $(N^2-1)/(48\pi^2 r^2)$.*

Proof. Positivity: for a test function f supported in a lift of a regular set, put $F = \sum_j f \circ R^{-j}$. By R-invariance of W_0, $W_0(\bar{F}, F) = N \sum_k W_0(\bar{f}, f \circ R^{-k}) = N\, W_N(\bar{f}, f)$, so $W_N(\bar{f}, f) \geq 0$. The wave equation and invariance are inherited termwise. For $x, x' \in V$ and $k \neq 0$ the spatial separation of x and $R^k x'$ is at least $2(r_p - \rho)s - 2\rho > 2\rho \geq |t - t'|$, so $x - R^k x'$ is spacelike and bounded away from the light cone, and $W_0(x, R^k x') = 1/(4\pi^2 (x - R^k x')^2)$ is smooth and real there. Symmetry and the coincidence value are as in Lemma .

par *Non-claim.* This supplies a state and its local Hadamard remainder for $\alpha = 1/N$; it is not a claim that this state is the physically preferred one on the cone. par

Theorem (Precise status of the transfer).

1. *For $\alpha = 1/N$, the cone-specific part of the construction (Lemma , the cross, HH and seagull terms, and the Ward computation of Theorem (b)) uses only: a smooth symmetric remainder harmonic (resp. a solution of the wave equation) in each variable, and the distributional equation of the flat parametrix. Each of these is supplied in the Lorentzian case by Proposition . What is not supplied is the flat Lorentzian renormalized product (a time-ordered extension of the W_0-built product across the light cone); this sheet does not construct it. So the Lorentzian product is conditional on that one flat datum.*

2. *For $\alpha \notin \{1/N : N \geq 2\}$ the image method does not apply: if α is irrational the rotation by $2\pi\alpha$ generates a dense subgroup of $SO(2)$, and if $\alpha = p/q$ in lowest terms with $p \geq 2$ the generated group is $\mathbb{Z}_q$, whose quotient has cone angle $2\pi/q \neq 2\pi\alpha$. A Green function (or state) for these α is not supplied by the inputs and is not constructed here.*

Proof. (a) Inspection of the proofs: Theorem (b) uses $\Delta H = 0$, symmetry of H, and $-\Delta G_0 = \delta$; the Lorentzian analogues are $\mathrm{Box}_x H_L = 0$, symmetry (Proposition), and the parametrix equation of the flat Feynman function, whose renormalized square is the missing flat datum. (b) Elementary: the subgroup generated by a rotation by $2\pi p/q$ ($\gcd(p, q) = 1$) is the cyclic group of order q; for irrational α it is infinite and dense.

par *Non-claim.* Part (b) is an obstruction to a method, not to existence: it does not say that no product exists for other deficits. The prescribed cone is not globally hyperbolic (I-B-G prop:nonGH), so general uniqueness theorems for Hadamard states are not available and are not invoked. par

v2: sharpened non-determination

Theorem (b) showed that the translation-invariant admissible extensions contain a line. We now compute the exact dimension and the exact minimal fixing datum. Let $\mathrm{Adm}_{\mathrm{inv}}(u^\circ, \omega)$ be the members of $\mathrm{Adm}(u^\circ, \omega)$ invariant under translations along W, and $\mathrm{Adm}_{\mathrm{cov}}$ those that are in addition covariant under $SO(1,1) \times SO(2)$ (boosts along W, rotations about W).

Theorem (Exact dimension and minimal datum). *Let K_ω be the space of symmetric tensor distributions on $\mathbb{R}^{1,3}$ supported in W, invariant under translations along W, conserved, with $\mathrm{sd}_W \leq \omega$; let $K_\omega^{\mathrm{cov}} \subseteq K_\omega$ be its covariant part.*

1. *$K_\omega = \{\sum_{|\beta| \leq \omega - 2} c_\beta^{bc} \partial_\perp^\beta \delta(x_\perp) : c$ constant, $\partial_b D^{bc} = 0\}$, and $\dim K_\omega = 25, 45, 71$ for $\omega = 4, 5, 6$; $\dim K_\omega^{\mathrm{cov}} = 3, 3, 5$. At $\omega = 4$, K_4^{cov} is spanned by $\eta_\parallel^{bc}\delta(x_\perp)$, $\eta_\parallel^{bc}\Delta_\perp\delta(x_\perp)$ and $(\partial_\perp^b\partial_\perp^c - \delta_\perp^{bc}\Delta_\perp)\delta(x_\perp)$, and $\Theta(1)$ is a combination of the last two.*

2. *If $\mathrm{Adm}_{\mathrm{inv}}(u^\circ, \omega) \neq \varnothing$, it is an affine space $U + K_\omega$ of dimension $\dim K_\omega$; likewise $\mathrm{Adm}_{\mathrm{cov}} = U + K_\omega^{\mathrm{cov}}$ for covariant U.*

3. *(Minimal datum) A family of n linear functionals* $\Lambda_1, \ldots, \Lambda_n$ *on* $U + K_\omega$ *determines the member uniquely from its values iff* $\Lambda = (\Lambda_i)$ *restricted to* K_ω *(as linear part) is injective. Hence exactly* $\dim K_\omega$ *real numbers are necessary and some* $\dim K_\omega$ *real numbers suffice (and* $\dim K_\omega^{\mathrm{cov}}$ *in the covariant class).*

4. *Without translation invariance no finite family of continuous linear functionals determines the member of* $\mathrm{Adm}(u^\circ, \omega)$, $\omega \geq 4$.

Proof. (a) A translation-invariant distribution supported in W is, by the structure theorem (as in I-B-G-Q prop:v5-codim2(i)), $\sum a_\beta(y) \otimes \partial_\perp^\beta \delta$ with translation-invariant, hence constant, a_β; tangential derivatives of constants vanish. The degree bound gives $|\beta| \leq \omega - 2$. Conservation is the linear system $\sum_{b \in \{2,3\}} k_b D^{bc}(k_\perp) = 0$ on the polynomial symbols; its solution spaces were computed by exact rank computation (C-Q-15). By hand at $\omega = 4$: the $\| \, \|$ block is unconstrained ($3 \times 6 = 18$), each mixed column is $q(k_3, -k_2)$ with $\deg q \leq 1$ ($2 \times 3 = 6$), and the $\perp \, \perp$ block is $q(k_3^2, -k_2 k_3; -k_2 k_3, k_2^2)$ with q constant (1): 25. Covariance leaves the three displayed generators (C-Q-17); $\Theta(1)$ has $\| \, \|$ block $-\eta_\| \Delta_\perp \delta$ and $\perp \, \perp$ block $(\partial\partial - \delta\Delta)_\perp \delta$ (C-Q-16). (b) Two members differ by an element of K_ω, and adding an element of K_ω preserves every condition, the degree bound because sd of a sum is at most the maximum. (c) Values of Λ on $U + K$ determine the point iff $\Lambda(U + k) = \Lambda(U + k') \Rightarrow k = k'$, i.e. iff $\Lambda|_K$ (linear part) is injective; an injective linear map from a d-dimensional space needs $n \geq d$, and coordinates in a basis give $n = d$. (d) $\Theta(\mathscr{D}'(W)) \subseteq$ direction space (Lemma) is infinite-dimensional; a finite family of functionals has a kernel of finite codimension in it, which is nonzero.

par *Non-claim.* The theorem counts exactly how many real normalization data the typed inputs leave free; it does not supply any of them, and it does not assert that the inherited coefficient lies in Adm. It corrects the v1 remark that ``one real number remains free" (see Appendix): that count is the scalar count of I-B-G-Q prop:v5-codim2(ii) at minimal degree, not the tensor count. No quantum coefficient is admitted. par

v2: independent re-verification of v1 claims

1. *Off-diagonal product.* Re-verified numerically by finite differences at a generic point (C-Q-14), independently

of the symbolic C-Q-1.

2. *Ward identity for $\Pi^{(v)}$.* Re-verified in momentum space: the symbol $\delta k^2 - kk$ is transverse (C-Q-4), and Theorem (c) shows that $\widehat{\Pi}^{(v)}$ is that symbol times a scalar distribution, so transversality holds for every v.

3. *Running coefficient and dilation sign.* Re-derived through the Fourier transform of $|x|^{-4+2\varepsilon}$ (C-Q-12, C-Q-13), with the same value $-1/(24\pi^2)$ per $\log v$ and the same sign as v1.

4. *Non-determination.* v1's line $U + t\Theta(1)$ is confirmed and embedded in the exact affine space of Theorem .

v3: the flat Lorentzian product is not determined by the Euclidean data

Theorem (a) left one flat datum missing: a Lorentzian renormalized product (a time-ordered extension). We show that the Euclidean product and every property proved for it do not determine that datum, and we write down one explicit declared datum with all those properties. Work on $\mathbb{R}^{1,3}$ with $\eta = \mathrm{diag}(-1,1,1,1)$, momentum k, $q(k) = \eta^{ab}k_a k_b$ (so $q > 0$ is spacelike, the region reached from the Euclidean section by $k_4 = \mathrm{i}k_0$), $P_{ab}(k) = \eta_{ab}q - k_a k_b$, $a = 1/(48\pi^2)$, and $c \in \mathbb{R}$ the subtraction constant of Theorem (c) ($c = c_\star/\pi^2$).

Definition (Lorentzian comparison family). For $(\alpha,\beta) \in \mathbb{R}^2$ and $v > 0$ let

$$\widehat{\mathfrak{P}}^{\alpha,\beta}_v(k) = a\,P(k)[\log\frac{|q(k)|}{v^2} - c + \mathrm{i}\pi\,\theta(-q(k))(\alpha + \beta\,\mathrm{sgn}\,k_0)].$$

The member $(\alpha,\beta) = (-1,0)$ is $aP[\log((q - \mathrm{i}0)/v^2) - c]$; $(1,0)$ is $aP[\log((q + \mathrm{i}0)/v^2) - c]$; $(0,\mp 1)$ is $aP[\log((q \mp \mathrm{i}0\,\mathrm{sgn}\,k_0)/v^2) - c]$.

Lemma (Each member is a tempered, covariant, transverse, running distribution). *For every (α,β): $\widehat{\mathfrak{P}}^{\alpha,\beta}_v$ is a locally integrable function of polynomial growth, hence a tempered distribution; it is symmetric and invariant under the proper orthochronous Lorentz group (as a tensor); $k^a\widehat{\mathfrak{P}}^{\alpha,\beta}_{v,ab} = 0$; $v\partial_v\widehat{\mathfrak{P}}^{\alpha,\beta}_v = -\frac{1}{24\pi^2}P$; and $\widehat{\mathfrak{P}}^{\alpha,\beta}_v(\lambda k) = \lambda^2(\widehat{\mathfrak{P}}^{\alpha,\beta}_v(k) + \frac{1}{24\pi^2}\log\lambda\,P(k))$ for $\lambda > 0$. On the spacelike region $q > 0$ all members equal $aP[\log(q/v^2) - c]$,*

the Euclidean form factor of Theorem (c) under $k_E^2 \mapsto q$, $\delta k_E^2 - k_E k_E \mapsto P$.

Proof. Local integrability. $\log|q|$ is integrable near the cone $q = 0$ away from $k = 0$ (there q is a coordinate and $\log|s|$ is integrable in one variable), and near $k = 0$ it is $O(\log|k| + |\log|q/|k|^2||)$, integrable on bounded sets in $\mathbb{R}^4$ (polar coordinates: the angular factor $\log|q(\omega)|$ is integrable on S^3 for the same reason). The bracket grows logarithmically, so P ×bracket is tempered. *Covariance.* q, P, $\theta(-q)$ and, on $q < 0$, $\operatorname{sgn} k_0$ are invariant under proper orthochronous transformations. *Transversality.* $k^a P_{ab} = k_b q - q k_b = 0$ (C-Q-21). *Running.* Only $-\log v^2$ depends on v; multiply by a: $-2a = -1/(24\pi^2)$ (C-Q-21). *Scaling.* $q(\lambda k) = \lambda^2 q(k)$, θ and sgn are scale invariant, $P(\lambda k) = \lambda^2 P(k)$, and $2a\log\lambda = \log\lambda/(24\pi^2)$. *Spacelike region.* $\theta(-q) = 0$ there.

Theorem (Euclidean data do not determine the Lorentzian product). *Let* $\mathscr{R}$ *be the following list of properties of a tempered symmetric tensor distribution* $\widehat{\mathfrak{P}}_v$ *on* $\mathbb{R}^{1,3}$*: (L1) proper orthochronous Lorentz covariance; (L2) transversality; (L3) the running* $v\partial_v \widehat{\mathfrak{P}}_v = -\frac{1}{24\pi^2} P$*; (L4) the scaling relation of Lemma ; (L5) agreement on the open spacelike region* $q > 0$ *with the continued Euclidean form factor. Then the set of distributions with* $\mathscr{R}$ *contains the two-parameter family of Definition , whose members are pairwise distinct; distinct members differ by*

$$\widehat{\mathfrak{P}}_v^{\alpha,\beta} - \widehat{\mathfrak{P}}_v^{\alpha',\beta'} = \mathrm{i}\pi a\, P\, \theta(-q)((\alpha - \alpha') + (\beta - \beta')\operatorname{sgn} k_0),$$

which is not a polynomial, so the difference is not a point-supported (local) counterterm in position space. In particular the Feynman member $(-1, 0)$ *and the member* $(1, 0)$ *differ by* $\frac{\mathrm{i}}{24\pi} P\,\theta(-q) \neq 0$ *(C-Q-20). Hence the Euclidean product of Theorem , with every property proved for it in this sheet, does not determine a flat Lorentzian product, even modulo local counterterms.*

Proof. By Lemma , every member has (L1)–(L5). The displayed difference is immediate from the definition; it is nonzero on the open timelike region whenever $(\alpha, \beta) \neq (\alpha', \beta')$ (take $k_0 > 0$ and $k_0 < 0$). A distribution is supported at 0 iff its Fourier transform is a polynomial (Paley–Wiener–Schwartz); a function that vanishes on an open set without vanishing identically is not a polynomial. For the Feynman pair, $\mathrm{i}\pi a \cdot (-2)$ in absolute value gives

$2\pi a = 1/(24\pi)$.

par *Non-claim.* This is a non-determination statement about the listed data, adopted from the user draft D5 (Theorem Q-nontransfer) and sharpened to a two-parameter family and to non-locality of the differences. It does not say that the members are all time-ordered products of the declared interaction; a time-ordering prescription is additional data, and it is precisely the datum that is missing. No quantum coefficient is admitted. par

Proposition (One explicit declared Lorentzian datum). *Declare the flat Lorentzian product to be the inverse Fourier transform* $\mathfrak{P}_v^F$ *of the Feynman member* $\widehat{\mathfrak{P}}_v^{-1,0} = aP[\log((q - \mathrm{i}0)/v^2) - c]$. *Then* $\mathfrak{P}_v^F$ *is symmetric, Lorentz covariant, conserved (*$\partial^a \mathfrak{P}^F_{v,ab} = 0$*), satisfies* $\mathfrak{P}^F_{v'} - \mathfrak{P}^F_v = -\frac{1}{24\pi^2}\log\frac{v'}{v}(\partial\partial - \eta\mathrm{Box})\delta_0$, *and* $\mathfrak{P}^F_v(\lambda\,\cdot) = \lambda^{-6}(\mathfrak{P}^F_v - \frac{1}{24\pi^2}\log\lambda\,(\partial\partial - \eta\mathrm{Box})\delta_0)$. *Within the class of Theorem with the same timelike boundary values, it is unique up to the scale v.*

Proof. Covariance, symmetry and transversality are Lemma ; transversality in momentum space is conservation in position space. The v-difference is $-2a\log(v'/v)P$, and with $\partial \to \mathrm{i}k$ the operator $\partial\partial - \eta\mathrm{Box}$ has symbol $-kk + \eta q = P$, so P is the transform of $(\partial\partial - \eta\mathrm{Box})\delta_0$ and $2a = 1/(24\pi^2)$. The scaling relation is the Fourier transform of the momentum relation of Lemma ($\mathbb{R}^4$: a factor λ^{-4} from the transform and λ^{-2} from P). Uniqueness: two members with the same values on $q < 0$ and on $q > 0$ (the latter by (L5)) agree off the cone $q = 0$, a null set, hence agree as locally integrable functions, up to the choice of v and c; changing c is changing v.

par *Non-claim.* The datum is declared, not derived: that $\mathfrak{P}_v^F$ restricts away from the light cone of the origin to the time-ordered pointwise product of the Feynman functions of the declared free field is the standard Wick-rotation identity, which this sheet does not prove. Therefore Theorem (a) is now conditional on one *explicit* distribution and one identification, instead of on an unnamed datum; it is not made unconditional. No Lorentzian product of the inherited theory is claimed, and the product runs (RG-open). No quantum coefficient is admitted. par

v3: the finite-image method has an exact boundary

Theorem (Finite rotational images). *For $0 < \alpha \le 1$, the two-dimensional cone of total angle $2\pi\alpha$ is isometric to a quotient $\mathbb{R}^2/G$ of the Euclidean plane by a finite group G of rotations about the origin iff $\alpha = 1/N$ for some $N \in \mathbb{N}$. Consequently the image construction of Section (finite sums over G of the flat Green function or two-point function) produces the prescribed cone exactly for $\alpha \in \{1/N\}$ and for no other deficit.*

Proof. A finite subgroup G of $SO(2)$ is cyclic: the angles of its elements form a finite subgroup of $\mathbb{R}/2\pi\mathbb{Z}$, which is generated by its smallest positive element $2\pi/N$, $N = |G|$. The quotient of the plane by the rotation of order N is the cone with fundamental sector of angle $2\pi/N$. Cone angle is an isometry invariant (it is the length of the unit circle about the apex), so the cone of angle $2\pi\alpha$ arises iff $\alpha = 1/N$. For $\alpha = p/q$ in lowest terms the rotation by $2\pi\alpha$ generates $\mathbb{Z}_q$, with smallest positive angle $2\pi/q$, and the quotient angle is $2\pi/q \neq 2\pi\alpha$ when $p \ge 2$ (C-Q-22). For irrational α the generated group is infinite and dense, so not finite. $N = 1$ is the flat case.

par *Non-claim.* This is an obstruction to a *method*, not to existence: it does not say that no product, state or Green function exists for other deficits. Other constructions (spectral or Sommerfeld-type) are recorded in the I-B-VP series and are documentary, not premises. The prescribed cone is not globally hyperbolic (I-B-G prop:nonGH), so no general Hadamard uniqueness is invoked. The OpenAI catalogue records OpenAI rem:oai-p-090-04 (renormalized energy for planar Coulomb systems) and OpenAI rem:oai-p-079-01 (critical local smoothing) concern different renormalizations and estimates; documentary only. par

v3: the actual-lift defect is not determined by the typed inputs

Proposition (No detecting value or sign from the typing). *In the setting of Proposition , a real test φ supported in $U \subset \mathscr{W} \setminus Z$ with $\langle b, \varphi\rangle \neq 0$ would exclude solvability. The typed inputs (T0)–(T3), the conservation and degree conditions on corrections, the correction supports and the response map $\mathscr{L}$ do not determine $\langle b, \varphi\rangle$, its sign, or whether it vanishes.*

Proof. The first statement is Proposition . For the second, fix the correction space, Z, $\mathscr{L}$ and the typing. Choose a real distribution B with $\operatorname{supp} B \subset U$ and $\langle B, \varphi\rangle = 1$ (e.g. $B = \varphi/\|\varphi\|_{L^2}^2$). The problems with defects $b = 0$, $b = B$ and

$b = -B$ share every listed datum, since none of these data constrains the preliminary lift's defect, and have detecting values 0, 1, −1. So neither the value nor the sign is a function of the listed data. The actual preliminary lift is not constructed in the inputs (I-B-G-Q thm:release(vi)), so b is not available to evaluate.

par *Non-claim.* This is the user draft D5's Proposition Q-defectgap, re-proved. It does not evaluate the actual-lift defect, and it draws no impossibility: Theorem already forbids inferring failure from the typing alone. The separator stays conditional. No quantum coefficient is admitted. par

v3: the normalization slots are unfilled in every degree

Theorem (All-degree count). *Let $m \geq 0$ be the maximal normal derivative order (transverse scaling bound $\omega = m + 2$). The space K_{m+2} of Theorem has*

$$\dim K_{m+2} = d_m = 3\tfrac{m+2}{2} + 2\tfrac{m+1}{2} + \tfrac{m}{2} = 3m^2 + 5m + 3, \qquad \dim K^{\mathrm{cov}}_{m+2} = 2\lfloor m/2 \rfloor + 1.$$

For $m = 2, 3, 4$ this gives $25, 45, 71$ and $3, 3, 5$, as in Theorem .

Proof. Write symbols $D^{bc}(u, v)$, $k_\perp = (u, v)$, polynomials of degree $\leq m$; translation invariance along W kills tangential momenta, so conservation is $\sum_{b\perp} k_b D^{bc} = 0$ for each c. *Tangential block:* unconstrained, three entries: $3\frac{m+2}{2}$. *Mixed:* for each of the two tangential c, (D^{uc}, D^{vc}) satisfies $uA + vB = 0$; since u, v are coprime, $(A, B) = r(v, -u)$ with $\deg r \leq m - 1$: $2\frac{m+1}{2}$. *Transverse block:* symmetric with both columns divergence-free: the first gives $(D^{uu}, D^{vu}) = r_1(v, -u)$, the second $(D^{uv}, D^{vv}) = r_2(v, -u)$; symmetry $D^{vu} = D^{uv}$ gives $-ur_1 = vr_2$, so $r_1 = vr$, $r_2 = -ur$ and $D_{\perp\perp} = r \begin{pmatrix} v^2 & -uv \\ -uv & u^2 \end{pmatrix}$ with $\deg r \leq m - 2$: $\frac{m}{2}$. Sum: $\frac{1}{2}[3(m+2)(m+1) + 2m(m+1) + m(m-1)] = 3m^2 + 5m + 3$. *Covariant part.* $SO(1,1)$-invariant symmetric tensors on the tangential plane are multiples of $\eta_\parallel$, and there is no invariant tangential vector, so the mixed block vanishes and the tangential block is $\eta_\parallel A(u^2 + v^2)$, $\deg A \leq m$: $\lfloor m/2 \rfloor + 1$ generators. $SO(2)$-covariant symmetric transverse tensors are $f\delta_\perp + gk_\perp k_\perp + h\,(k_\perp \otimes \varepsilon k_\perp + \varepsilon k_\perp \otimes k_\perp)$ with f, g, h functions of $u^2 + v^2$; the divergence is $(f + gr^2)k_\perp + hr^2 \varepsilon k_\perp$, so $h = 0$ and $f = -gr^2$, giving $g\,(kk - r^2\delta)_\perp$, which is $-g$ times the displayed matrix: $\lfloor m/2 \rfloor$

generators (degree $\le m$). Total $2\lfloor m/2 \rfloor + 1$. (C-Q-18 recomputes d_m, $m \le 7$, by floating-point SVD of the conservation system, a method different from v2's exact rank; C-Q-19 the covariant formula.)

par *Non-claim.* A count of the free normalization directions; none is fixed, and no coefficient is admitted. par

Corollary (No typed input fills a slot). *If* $\mathrm{Adm}_{\mathrm{inv}}(u^{\circ}, m+2) \neq \varnothing$, *then for every subset* $\mathscr{S}$ *of the typed inputs* {*agreement off W, translation invariance, conservation, degree bound, (T1)–(T3)*}, *every coordinate of the* d_m *-dimensional direction space varies freely on the fibre of* $\mathscr{S}$*; the same holds for the* d_m^{cov} *covariant coordinates. A family of external linear data fixes the member iff it is injective on* K_{m+2}, *so at least* d_m *(resp.* d_m^{cov}*) real numbers must be supplied from outside the inputs.*

Proof. The full set of typed inputs is preserved by adding any $k \in K_{m+2}$ (Theorem (b)); (T1)–(T3) concern the ambiguity shape and the incidence space and do not constrain k (Theorem (c)). A subset of the inputs is weaker than the full set, so its fibre contains $U + K_{m+2}$. The injectivity statement is Theorem (c).

par *Non-claim.* The count says how many normalization numbers are missing; it supplies none, and it does not claim that the inherited coefficient lies in Adm. The coefficient is not admitted; the certificate remains RG-open. par

v3: re-verification of v2 claims

1. *Theorem (*$25, 45, 71; 3, 3, 5$*).* Re-derived by hand in all degrees (Theorem) and recomputed by floating-point SVD (C-Q-18), different from v2's exact rank (C-Q-15). Confirmed.

2. *Theorem* . Its hypothesis (T0) was wrong for the benchmark (Appendix); the theorem holds under the corrected NSS hypothesis, with the proof adjusted.

3. *Running.* The coefficient $-1/(24\pi^2)$ is recovered independently in the Lorentzian family (C-Q-21).

v4: the declared Feynman datum is a unitary Epstein–Glaser product

Version 3 declared the flat Lorentzian datum $\mathfrak{P}_v^F$ (Proposition). It left unproved that this datum is the time-ordered

product, and its non-claim omitted a factor i. Here we compute the datum off the origin, identify it with an Epstein–Glaser (EG) extension, and state exactly which additional datum makes it unique. Conventions are those of Section : $\eta = \mathrm{diag}\,(-1,1,1,1)$, $x^2 = \eta_{ab}x^a x^b$, $a = 1/(48\pi^2)$, $c_0 = -1/(48\pi^4)$, and $\Delta_F(x) = 1/(4\pi^2(x^2 + \mathrm{i}0))$, the vacuum time-ordered two-point function of the massless scalar in this signature.

Lemma (The datum off the origin). *Let* $C(\lambda) = \frac{\mathrm{i}}{\pi^2}4^\lambda \Gamma(\lambda+2)/\Gamma(-\lambda)$. *Off the origin,* $\mathscr{F}^{-1}[(q-\mathrm{i}0)^\lambda] = C(\lambda)(x^2+\mathrm{i}0)^{-\lambda-2}$, $C(-1) = \mathrm{i}/(4\pi^2)$, *and* $\mathscr{F}^{-1}[\log(q-\mathrm{i}0)] = -\frac{\mathrm{i}}{\pi^2}(x^2+\mathrm{i}0)^{-2}$. *Consequently, on* $\mathbb{R}^{1,3}\setminus\{0\}$,

$$\mathfrak{P}_v^F = \mathrm{i}\,T^\circ, \qquad T^\circ := c_0(\partial\partial - \eta\mathrm{Box})(x^2+\mathrm{i}0)^{-2} = 2(\partial\Delta_F\,\partial\Delta_F - \Delta_F\,\partial\partial\Delta_F).$$

Proof. The formula for $(q-\mathrm{i}0)^\lambda$ is the Lorentzian continuation of the Euclidean Riesz transform $\widehat{|x|^{-4+2\varepsilon}}$ used in Theorem (c) (Gel'fand–Shilov, Ch. III 2). At $\lambda = -1$ it reproduces Δ_F from $\mathscr{F}[\Delta_F] = 1/(q-\mathrm{i}0)$ in this normalization (C4-Q-1). Since $C(0) = 0$ with $C(\lambda)/\lambda \to -\mathrm{i}/\pi^2$, differentiating at $\lambda = 0$ gives the logarithm off the origin; the constant term and $\log v^2$ are supported at 0. Since $\partial \to \mathrm{i}k$ turns $\partial\partial - \eta\mathrm{Box}$ into P, off the origin $\mathfrak{P}_v^F = a(\partial\partial - \eta\mathrm{Box})(-\frac{\mathrm{i}}{\pi^2})(x^2+\mathrm{i}0)^{-2}$, and $-\mathrm{i}a/\pi^2 = \mathrm{i}c_0$ (C4-Q-3). The second form of T° is the identity of C4-Q-2 between rational functions of x, applied to boundary values from the same side $x^2 + \mathrm{i}0$, where the products are defined off 0.

par *Non-claim.* An off-origin identity between explicit distributions; it is not an extension across the origin and not a statement about the inherited coefficient. No quantum coefficient is admitted. par

By Wick's theorem, T° is the off-origin vacuum time-ordered product of the conserved current pair, with the real normalization of Lemma carried to Lorentzian signature. It is the Lorentzian counterpart of Π°. We take this standard identification as cited (EG; Brunetti–Fredenhagen), not re-proved. Put $T_v^F := -\mathrm{i}\,\mathfrak{P}_v^F$, an extension of T° to $\mathbb{R}^{1,3}$.

Theorem (EG extensions, and the extra datum).

1. *The tensor distributions on* $\mathbb{R}^{1,3}$ *that extend* T°*, are symmetric, Lorentz covariant and conserved, and have scaling degree* ≤ 6 *at* 0*, are exactly*

$$T_v^F + \zeta(\partial\partial - \eta\mathrm{Box})\delta_0, \qquad \zeta \in \mathbb{C}.$$

Fixing v, this is one complex parameter. In momentum space the family reads $-\mathrm{i}aP[\log(q - \mathrm{i}0) - w]$ *with* $w = \log v^2 + c - \mathrm{i}\zeta/a \in \mathbb{C}$*. Changing v moves* $\mathrm{Re}\, w$*;* $\mathrm{Re}\,\zeta$ *moves* $\mathrm{Im}\, w$*.*

2. *Let* $W(x) = \langle j(x)j(0)\rangle$ *be the (conserved, covariant) Wightman two-point distribution of the current pair, with the same normalization as* T°*. Impose the two-point unitarity condition* $T + \overline{T} = W + W(-\,\cdot)$*, which holds off the origin for* T° *because the anti-time-ordered product is the adjoint of the time-ordered one. Within (a), unitarity holds iff* $\mathrm{Re}\,\zeta = 0$*. The unitary extensions are then exactly* $\{T_v^F : v > 0\}$*, one for each scale.*

3. *Hence the declared datum of Proposition is* $\mathfrak{P}_v^F = \mathrm{i}\,T_v^F$*, where* T_v^F *is the unitary EG time-ordered product at scale v. The datum is unique up to v. The extra datum beyond the EG axioms (extension, covariance, symmetry, conservation, degree) is precisely unitarity, which removes one real parameter. v is not fixed, and the product stays RG-open.*

Proof. (a) Two such extensions differ by a distribution supported at 0 of scaling degree ≤ 6, hence by $\sum_{|\gamma|\le 2} c_\gamma \partial^\gamma \delta_0$. Covariance and symmetry leave $A\eta\delta_0 + B\partial\partial\delta_0 + C\eta\mathrm{Box}\,\delta_0$, with symbol $A\eta - Bkk - C\eta q$. Conservation forces $A = 0$ and $B = -C$ (C4-Q-5), i.e. $\zeta(\partial\partial - \eta\mathrm{Box})\delta_0$ with $\zeta = B$. T_v^F is itself such an extension: it is covariant, symmetric and conserved by Lemma , and its degree is 6 with logarithm, as Proposition shows. The momentum form follows from $(\partial\partial - \eta\mathrm{Box})\delta_0 \mapsto P$. (b) For T in (a), $D = T + \overline{T} - W - W(-\,\cdot)$ vanishes off 0, is covariant, symmetric and conserved, and has degree ≤ 6. So $D = r(\partial\partial - \eta\mathrm{Box})\delta_0$, and r is real because D is real. Fourier transform with $\mathscr{F}[\overline{T}](k) = \overline{\widehat{T}(-k)}$:

$$\mathscr{F}[T_v^F + \overline{T_v^F}](k) = -\mathrm{i}aP[\log(q - \mathrm{i}0) - \log(q + \mathrm{i}0)] = -2\pi a\, P\,\theta(-q)$$

(C4-Q-4). This vanishes on the spacelike region $q > 0$. $\widehat{W}$ is supported in the closed forward cone (spectrum

condition) and $\widehat{W(-\cdot)}$ in the backward cone, so both vanish on $q > 0$. On $q > 0$ we have $P \neq 0$, so $r = 0$ for T_v^F, and for $T = T_v^F + \zeta(\cdots)$ we get $\widehat{D} = 2\mathrm{Re}\,\zeta P$, which vanishes iff $\mathrm{Re}\,\zeta = 0$. Finally $T_{v'}^F - T_v^F = \frac{\mathrm{i}}{24\pi^2}\log\frac{v'}{v}(\partial\partial - \eta\mathrm{Box})\delta_0$ by Proposition , and these exhaust the imaginary ζ. (c) Combine Lemma with (a) and (b).

par *Non-claim.* The off-origin identification of T° with the time-ordered product, and the spectrum condition for W, are cited standard facts, not proved here. The theorem supplies the flat datum of Theorem (a). It does not re-prove the Lorentzian cone Ward identity for $\alpha = 1/N$ with that datum, does not construct the Lorentzian cone product, and fixes no scale. It is a statement about a free charged scalar, not about the inherited coefficient. No quantum coefficient is admitted. par

v4: the Euclidean cone product for every deficit

Theorem bounds the image *method*. Here a different construction, the closed-form mode sum (of Sommerfeld type), supplies the Green function and the product for every cone angle $2\pi\alpha$, $\alpha > 0$. In developed coordinates the Euclidean section is $\mathrm{d}\tau^2 + \mathrm{d}r^2 + r^2\,\mathrm{d}\psi^2 + \mathrm{d}z^2$ with $\psi \sim \psi + 2\pi\alpha$. Put $\cosh u = (r^2 + r'^2 + \Delta\tau^2 + \Delta z^2)/(2rr')$ $(u \geq 0)$ and $\Delta\psi = \psi - \psi'$.

Definition (Cone Green function).

$$G_\alpha(x, x') = \frac{\sinh(u/\alpha)}{8\pi^2\alpha\, rr' \sinh u\,(\cosh(u/\alpha) - \cos(\Delta\psi/\alpha))}, \qquad H_\alpha = G_\alpha - G_1,$$

where G_1 is computed with the developed angle difference in $(-\pi\alpha, \pi\alpha]$. A *regular ball* is a ball V of radius ρ about a point at distance r_p from the axis, with $\rho < r_p \sin(\pi\min\{\alpha, \frac{1}{2}\})$.

Lemma (G_α is the Green function; H_α is smooth). *For every $\alpha > 0$:*

1. $G_\alpha = \frac{1}{8\pi^2\alpha rr' \sinh u}\sum_{n\in\mathbb{Z}} e^{-|n|u/\alpha} e^{\mathrm{i}n\Delta\psi/\alpha}$*; G_α is real, symmetric, $2\pi\alpha$-periodic in $\Delta\psi$, and harmonic in each variable off the diagonal.*

2. $G_1(x, x') = 1/(4\pi^2|x - x'|^2)$ *in the developed chart. For $\alpha = 1/N$, G_α equals the image sum G_N of Section (C4-Q-8).*

3. *On $V \times V$, for a regular ball V, H_α is smooth, real, symmetric and harmonic in each variable, $-\Delta_x G_\alpha(\cdot, x') = \delta_{x'}$, and*

$$H_\alpha(x,x) = \frac{\alpha^{-2} - 1}{48\pi^2 r(x)^2}.$$

Proof. (a) The sum is the Poisson-kernel identity $\sum_n e^{-|n|t} e^{in\theta} = \sinh t/(\cosh t - \cos\theta)$. Reality, symmetry and periodicity are visible. Harmonicity: write a mode as $A\, e^{-\nu u} e^{i\nu\psi}$ with $A = (rr' \sinh u)^{-1}$ and $\nu = |n|/\alpha$ (the sign of the ψ-exponent is immaterial). Its Laplacian in x is $e^{-\nu u + i\nu\psi}$ times a quadratic polynomial $Q(\nu)$:

$$Q(\nu) = \Delta A - \nu(2\nabla A \cdot \nabla u + A\Delta u) + \nu^2 A(|\nabla u|^2 - r^{-2}).$$

C4-Q-6 verifies $|\nabla u|^2 = r^{-2}$ and $\Delta A = 0$ exactly, and $2\nabla A \cdot \nabla u + A\Delta u = 0$ at 50 digits. So $Q \equiv 0$ and every mode is harmonic for every real ν. The series converges with all derivatives for $u > 0$, and at $u = 0$, $\Delta\psi \neq 0$ it is the closed form. Independently, C4-Q-9 checks $\Delta_x G_\alpha = 0$ by 40-digit differentiation for $\alpha = 0.37, 0.8, 1.6$. (b) $|x - x'|^2 = 2rr'(\cosh u - \cos\Delta\psi)$, and $\alpha = 1$ gives $1/(8\pi^2 rr'(\cosh u - \cos\Delta\psi))$. The image identity is C4-Q-8. (c) With $f_\alpha = \sinh(u/\alpha)/(\alpha \sinh u(\cosh(u/\alpha) - \cos(\Delta\psi/\alpha)))$, $H_\alpha = (f_\alpha - f_1)/(8\pi^2 rr')$. Near the diagonal, $\cosh a - \cos b = \frac{a^2 + b^2}{2}(1 + O(a^2 + b^2))$, and the leading singular parts of f_α and f_1 are both $2/(u^2 + \Delta\psi^2)$. So $f_\alpha - f_1$ is bounded near the diagonal, with direction-independent limit $(1 - \alpha^2)/(6\alpha^2)$ (C4-Q-7). Off the diagonal, on $V \times V$, H_α is real-analytic and harmonic in each variable, hence annihilated by the Laplacian of $\mathbb{R}^8$. The diagonal has codimension 4 in $\mathbb{R}^8$, hence zero capacity, and bounded harmonic functions extend harmonically across it. So H_α is smooth on $V \times V$. Harmonicity in each variable and symmetry pass to the extension by continuity. $-\Delta G_1 = \delta$ on V because V is a flat ball. The coincidence value is $(1 - \alpha^2)/(6\alpha^2) \cdot (8\pi^2 r^2)^{-1}$.

par *Non-claim.* A Euclidean Green function on regular balls; it is not a Lorentzian two-point function or a state, and no positivity is claimed for $\alpha \notin \{1/N\}$. par

Theorem (Cone product for every α). *For every $\alpha > 0$, every regular ball V and $\nu > 0$, define $\Pi_\alpha^{(\nu)}$ by Definition with H_α in place of H. Then (a)–(e) of Theorem hold verbatim. In particular, the seagull coefficient is forced to be*

$-2H_\alpha(x,x) = -(\alpha^{-2}-1)/(24\pi^2 r^2)$, *and the running is* $-\frac{1}{24\pi^2}(\partial\partial - \delta\Delta)\delta$, *independent of* α.

Proof. The proof of Theorem uses only: H smooth, real, symmetric and harmonic in each variable on $V \times V$; $-\Delta G_0 = \delta$; and the flat product. Lemma (c) supplies these for H_α.

par *Non-claim.* This is Euclidean only. The Lorentzian cone product for $\alpha \notin \{1/N\}$ is not constructed: a positive Lorentzian two-point function with the remainder H_α is not supplied here. Theorem remains true as a statement about the image method; the obstruction it records is to that method, and this section confirms that it is not an obstruction to existence. The closed form of G_α and the coincidence value are classical in the cosmic-string literature (documentary comparison); the proof here is self-contained. This is a product of a free field, not the inherited coefficient. No quantum coefficient is admitted. par

v4: the defect of a lift on a declared model

Proposition (Sign and size of the defect of the naive lift). *On the declared model of Theorem , let the naive lift be* $L_\alpha^{(v)} = \Pi^{(v)}(x-x') - X_{\mathrm{cr}} - X_{HH}$ *(built with* H_α*, without the seagull). Its Ward defect is*

$$b = \partial_x^\mu L^{(v)}_{\alpha,\mu\nu} = 2H_\alpha(x',x')\,\partial_\nu\delta(x-x'), \qquad 2H_\alpha(x',x') = \frac{\alpha^{-2}-1}{24\pi^2 r(x')^2}.$$

The coefficient is positive for $0<\alpha<1$ *(positive deficit), zero for* $\alpha = 1$*, and negative for* $\alpha > 1$*. At* $\alpha = 1/2$*,* $r = 1$ *it equals* $1/(8\pi^2)$ *(C4-Q-10). The defect is supported on the diagonal. So Proposition is silent:* $b|_U = 0$ *for every* U *avoiding the coincidence diagonal. Moreover* $-b \in \operatorname{im}\mathscr{L}$*: the correction* $-2H_\alpha(x,x)\delta_{\mu\nu}\delta(x-x')$ *removes it.*

Proof. This is the last display in the proof of Theorem (b) with $a = 0$, together with Lemma (c). The removal is Theorem (c) for H_α.

par *Non-claim.* This is the defect of a lift on a *declared* free-field model; it is not the defect of the actual preliminary lift of the inherited theory, which is not constructed in the inputs (Proposition stands). It shows that on this model the defect is local and removable, so the separator records no failure; it is no evidence about the inherited coefficient either way. No quantum coefficient is admitted. par

v4: independent re-verification of v3 claims

1. $d_m = 3m^2 + 5m + 3$. Recomputed by exact rational rank of the conservation system for $m = 0, \dots, 9$ (RV4-Q-1), and also by a generating function: $\sum_m d_m t^m = (3 + 2t + t^2)/(1 - t)^3$. This is a third method, after v2's exact rank at $m \le 4$ and v3's floating-point SVD. Confirmed.

2. i/(24π). By the Plemelj jump $\log(q - \mathrm{i}0) - \log(q + \mathrm{i}0) = -2\pi\mathrm{i}\,\theta(-q)$, the $(-1, 0)$ and $(1, 0)$ members differ by $aP \cdot (-2\pi\mathrm{i})\theta(-q)$, of modulus $2\pi a = 1/(24\pi)$ (C4-Q-4). Confirmed.

3. *Correction.* The v3 non-claim of Proposition said that Ψ_v^F ``restricts … to the time-ordered pointwise product". It restricts to i times that product (Lemma). The proposition itself is unaffected.

v5: the Lorentzian cone remainder for every opening

v4 constructed the Euclidean cone product for every $\alpha > 0$ (Theorem) and left the Lorentzian case open. D6 proposed a ``method obstruction" for all α; D7 proved that *termwise* substitution into the angular series fails (Lemma). Here we construct the Lorentzian *local* remainder for every α by analytic continuation of the closed form, a method different from both the image sum ($\alpha = 1/N$) and the Euclidean mode sum.

Lemma (Termwise angular summation fails). *Substituting* $u = \mathrm{i}v$, $0 < v < \pi$, *termwise into* $S_\alpha(u, \theta) = 1 + 2\sum_{n \ge 1} e^{-nu/\alpha}\cos n\theta$ *gives a divergent series for every* $\alpha > 0$ *and every real* θ.

Proof. The terms $2e^{-\mathrm{i}nv/\alpha}\cos n\theta$ have modulus $2|\cos n\theta|$. If $\cos n\theta \to 0$ then $\cos 2n\theta = 2\cos^2 n\theta - 1 \to -1$, a contradiction; so the terms do not tend to 0.

par *Non-claim.* An obstruction to one stated method only (D7, adopted), not to a Lorentzian Green function or product; Theorem uses a different method. par

Definition (Lorentzian remainder). For $\alpha > 0$ put

$$f_\alpha(u, \theta) = \frac{\sinh(u/\alpha)}{\alpha \sinh u\,(\cosh(u/\alpha) - \cos(\theta/\alpha))}, \qquad h_\alpha = f_\alpha - f_1.$$

On $\mathbb{R}^{1,3}_{\alpha}$ (metric $-\mathrm{d}t^2+\mathrm{d}r^2+r^2\,\mathrm{d}\psi^2+\mathrm{d}z^2$, $\psi\sim\psi+2\pi\alpha$), a *Lorentzian regular ball* is a geodesically convex coordinate ball V away from the axis such that for $x,x'\in V$ the developed angle $\theta=\Delta\psi$ and $u=\operatorname{arccosh} w$,

$$w=\frac{r^2+r'^2+\Delta z^2-\Delta t^2}{2rr'},$$

satisfy $|u|,|\theta|<\frac{\pi}{2}\min\{\alpha,1\}$ (for $w<1$, $u\in\mathrm{i}\mathbb{R}$). Put $H^L_\alpha(x,x')=h_\alpha(u,\theta)/(8\pi^2rr')$.

Theorem (Lorentzian local remainder and Ward-fixed product for every α). *For every $\alpha>0$ and every Lorentzian regular ball V:*

1. *h_α extends holomorphically to the polydisc $|u|,|\theta|<\frac{\pi}{2}\min\{\alpha,1\}$ in $\mathbb{C}^2$ and is even in u; hence H^L_α is a well-defined real, symmetric, smooth function on $V\times V$, independent of the choice of branch of* arccosh.

2. $\mathrm{Box}_x H^L_\alpha=\mathrm{Box}_{x'}H^L_\alpha=0$, *and H^L_α is the continuation $\tau\to\mathrm{i}t$ of the Euclidean remainder H_α of Definition .*

3. *For $\alpha=1/N$, H^L_α equals the Lorentzian image sum $\sum_{k=1}^{N-1}1/(4\pi^2\sigma_k)$.*

4. $H^L_\alpha(x,x)=\dfrac{\alpha^{-2}-1}{48\pi^2r^2}$.

5. *With the v4 convention $\mathscr{F}^{-1}[(q-\mathrm{i}0)^{-1}]=\mathrm{i}\Delta_F$, so that $\mathrm{Box}\,\Delta_F=\mathrm{i}\delta$, define X^F_{cr} and X_{HH} by the formulas of Definition with G_0 replaced by Δ_F and H by H^L_α, and*

$$T_\alpha=T^F_\nu(x-x')-X^F_{\mathrm{cr}}-X_{HH}+s(x)\,\eta_{\mu\nu}\delta(x-x').$$

Then $\partial^\mu_x T_{\alpha,\mu\nu}=(s(x')-2\mathrm{i}H^L_\alpha(x',x'))\partial_\nu\delta(x-x')$, so the Ward identity holds iff

$$s(x)=2\mathrm{i}H^L_\alpha(x,x)=\mathrm{i}\,\frac{\alpha^{-2}-1}{24\pi^2r^2}.$$

The running $v\partial_v T_\alpha=v\partial_v T^F_v$ is that of Theorem , independent of α.

Proof. (a) In the polydisc the only zeros of $\cosh(u/\alpha)-\cos(\theta/\alpha)$ and of $\cosh u-\cos\theta$ are on $u=\pm\mathrm{i}\theta$ (the other zeros have $|\mathrm{Im}\,u|\geq 2\pi\min\{\alpha,1\}-|\theta|$), and $\sinh u\neq 0$ for $0<|u|<\pi$, with f_α bounded as $u\to 0$ for $\theta\neq 0$. Along $u=\pm\mathrm{i}\theta$, $\theta\neq 0$, both f_α and f_1 have simple poles whose residue in u is $\pm 1/\sinh(\mathrm{i}\theta)$, independent of α (expand

$\cosh(u/\alpha) - \cos(\theta/\alpha)$ to first order at $u = \pm i\theta$: the factor $\alpha^{-1}\sinh(\pm i\theta/\alpha)$ cancels against the numerator; C5-Q-1). So h_α is holomorphic off $\{u = \theta = 0\}$, a set of complex codimension two, and extends by Hartogs' theorem. $f_\alpha(-u,\theta) = f_\alpha(u,\theta)$, so $h_\alpha = g_\alpha(u^2,\theta)$ with g_α holomorphic; $u^2 = \operatorname{arccosh}^2 w$ is real-analytic in w near $w = 1$, real for real w and branch-independent. The Taylor coefficients of g_α are real (both f are real on real arguments), so H^L_α is real; it is symmetric because w is and h_α is even in θ. (b) With $\tau = it$, w becomes the Euclidean $(r^2 + r'^2 + \Delta z^2 + \Delta\tau^2)/(2rr')$, and H^L_α becomes H_α (Definition), harmonic in each variable (Lemma). The function $\Delta_{\tau,r,\psi,z} H_\alpha$ is real-analytic in $\Delta\tau$ and vanishes; continuation to imaginary $\Delta\tau$ turns Δ into Box, so $\mathrm{Box}\, H^L_\alpha = 0$ (identity theorem; C5-Q-3). (c) For $\alpha = 1/N$ the Euclidean remainder is the image sum (Lemma); continue both sides (C5-Q-4). (d) At $x = x'$, $u = \theta = 0$ and $h_\alpha(0,0) = (\alpha^{-2} - 1)/6$, the Euclidean coincidence value (Proposition ; C5-Q-5, RV5-Q-3). (e) T^F_ν is conserved (Theorem). The Leibniz computation of Theorem with $\mathrm{Box}\, \Delta_F = i\delta$ (C5-Q-6) in place of $-\Delta G_0 = \delta$, and $\mathrm{Box}\, H^L_\alpha = 0$, gives $\partial^\mu X^F_{\mathrm{cr},\mu\nu} = 2i[(\partial'_\nu H - \partial_\nu H)(x',x')\delta + H(x',x')\partial_\nu\delta] = 2iH^L_\alpha(x',x')\partial_\nu\delta$ by symmetry, and $\partial^\mu X_{HH,\mu\nu} = 0$. Finally $\partial_\nu[s(x)\delta(x - x')] = s(x')\partial_\nu\delta(x - x')$. s and H^L_α do not depend on ν.

par *Non-claim.* A local construction on Lorentzian regular balls: no global propagator, no state, no positivity or unitarity for $\alpha \notin \{1/N\}$, and hence no identification with a time-ordered product (Theorem identified the flat datum using unitarity, which is not supplied here). The global diffracted contribution of I-B-ECHO (I-B-ECHO eq:Carslaw) arises for separations around the apex, outside these balls, and is not constructed. This is not the inherited coefficient; no quantum coefficient is admitted, and the product runs (RG-open). par

v5: the defect of a class of lifts

Definition (Smooth-remainder lift class). On a Euclidean regular ball V, let H be any smooth, real, symmetric bisolution ($\Delta_x H = \Delta_{x'} H = 0$), $a \in C^\infty(V)$, and $C_{\mu\nu}$ a diagonal-supported correction conserved in x. Put $L_{H,a,C} = \Pi^{(\nu)}(x - x') - X_{\mathrm{cr}} - X_{HH} + a(x)\delta_{\mu\nu}\delta(x - x') + C_{\mu\nu}$ (with X_{cr}, X_{HH} of Definition built from G_0 and H). The Lorentzian class is defined in the same way with Δ_F, Box, T^F_ν.

Theorem (Exactly removable local defect for the whole class). $\partial^{\mu}_{x} L_{H,a,C,\mu\nu} = (2H(x',x') + a(x'))\partial_{\nu}\delta(x-x')$*; it is diagonal-supported and removed by* $-(2H(x,x)+a(x))\delta_{\mu\nu}\delta(x-x')$*. In the Lorentzian class the defect is* $(a(x') - 2iH(x',x'))\partial_{\nu}\delta$*, removed in the same way. So the support separator (Proposition) detects nothing for these classes.*

Proof. $\Pi^{(\nu)}$ and C are conserved. Since $\Delta H = 0$, $\partial^{\mu} X_{\mathrm{cr},\mu\nu} = 2\Delta G_0\, \partial'_{\nu} H - 2H\, \partial'_{\nu}\Delta G_0 = -2\delta\, \partial'_{\nu} H - 2H\partial_{\nu}\delta$, using $\Delta G_0 = -\delta$ and $\partial'_{\nu}\delta(x-x') = -\partial_{\nu}\delta(x-x')$; moving the smooth coefficients to x', this is $2(\partial_{\nu} H - \partial'_{\nu} H)(x',x')\delta - 2H(x',x')\partial_{\nu}\delta$, and the first coefficient vanishes by symmetry. $\partial^{\mu} X_{HH} = 0$ by the same Leibniz rule and $\Delta H = 0$. $\partial_{\nu}[a\delta] = a(x')\partial_{\nu}\delta$. The Lorentzian case is Theorem (e) with general H.

par *Non-claim.* The defect is evaluated for a stated class of lifts (including the cone remainders, D7 UserDraft7 thm:Q-liftclass, adopted), not for the actual inherited preliminary lift, which is not constructed. Outside the class the typing supplies neither a detecting value nor a sign (correcting D6 UserDraft6 thm:Q-defectclass, whose ``sign determined'' is restricted to this class). The coefficient is not an order relation on $\partial\delta$. par

v5: running and the Oai282-A Ward certificate

Oai282-A thm:ward is a finite tested certificate: under Oai282-A ass:ward (symmetric insertion $\mathcal{T}^{r}$; improvement $t^{r} = \mathcal{T}^{r} + \frac{1}{3}(\partial\partial - \eta\mathrm{Box})\mathcal{L}^{r}$; finitely many conformal Killing fields X_a and tests; a linear contact assignment $\mathcal{C}_X$) it writes the Ward residual as $W = \sum_{\nu}\mathcal{E}_{\nu} + \mathcal{E}_{\mathrm{tr}} + \mathcal{K}_X$ evaluated on tests. Family 282 (OpenAI rem:oai-p-282-01) concerns unitary positive-energy theories with a scale-invariant vacuum and a local dilatation current.

Theorem (The running does not place the product outside the Ward certificate). *Transcribe Oai282-A ass:ward formally to the Euclidean product* ($\eta \to \delta$, Box $\to \Delta$) *with* $\mathcal{T} = \Pi^{(\nu)} = c_0 D u_{\nu}$, $D = \partial\partial - \delta\Delta$, $c_0 = -1/(48\pi^4)$.

1. *With* $\mathcal{L} = -3c_0 u_{\nu}$ *one has* $\Pi^{(\nu)} = -\frac{1}{3}D\mathcal{L}$*, so the improved insertion is* $t = 0$*; with* $\mathcal{P} = 0$, $\mathcal{S} = 0$ *all defects* $\mathcal{E}_{\nu}, \mathcal{E}_{\mathrm{tr}}$ *vanish and* $W = -\mathcal{C}_X(f)$*. With* $\mathcal{C} = 0$ *the certificate holds for every* v*.*

2. *The running is a contact term:* $(x\cdot\partial+4)u_v = 2\pi^2\delta$*, hence* $(x\cdot\partial+6)\Pi^{(v)} = -\frac{1}{24\pi^2}D\delta$*, independent of v.*

3. *Off coincidence* $\Pi^{\circ}_{\mu\nu} = \frac{1}{4\pi^4}\frac{\delta_{\mu\nu}-2x_\mu x_\nu/|x|^2}{|x|^6}$*, which transforms covariantly under inversion as a dimension-three vector two-point function; the scale witness is* $\operatorname{pair}\operatorname{tr}(v\partial_v\Pi^{(v)})\psi = \frac{1}{8\pi^2}$ *for* $\Delta\psi(0)=1$*. Hence the plan statement ``the running places the product outside Oai282-A thm:ward and Family 282'' is false as stated: the running excludes only hypotheses demanding literal scale independence of the extended distribution.*

Proof. (a) $\frac{1}{3}D(-3c_0u) = -c_0Du = -\Pi$. Then $\partial^\mu t = 0$, $\operatorname{tr} t = 0$; $\mathcal{E}_v = \partial\Pi - 0 = 0$ (Theorem); $\mathcal{E}_{\mathrm{tr}} = \operatorname{tr}\Pi - \Delta\mathcal{L} = -3c_0\Delta u + 3c_0\Delta u = 0$ (as $\operatorname{tr} D = -3\Delta$); $\mathcal{K}_X = -\mathcal{C}_X$; insert in Oai282-A thm:ward. (b) $\operatorname{pair}(x\cdot\partial+4)u_v\varphi = \langle u_v, -\partial\cdot(x\varphi)+4\varphi\rangle = \langle u_v, -x\cdot\partial\varphi\rangle = 2\pi^2\varphi(0)$, the last equality because $\int|x|^{-4}x\cdot\partial\varphi$ converges and equals $-2\pi^2\varphi(0)$ by integration in the radius (C5-Q-7); then $[x\cdot\partial, D] = -2D$ gives $(x\cdot\partial+6)Du = D(x\cdot\partial+4)u = 2\pi^2 D\delta$, and $2\pi^2 c_0 = -1/(24\pi^2)$. (c) $D|x|^{-4} = -12|x|^{-6}(\delta - 2xx/|x|^2)$ (C5-Q-8) times c_0; inversion covariance from $|\iota x - \iota y|^2 = |x-y|^2/(|x|^2|y|^2)$ and $J(\iota x-\iota y) = J(x)J(x-y)J(y)$ (C5-Q-8); $\operatorname{tr} D\delta = -3\Delta\delta$ and Theorem (a).

par *Non-claim.* (a) is a formal transcription with the trivial contact assignment; it is not an application of the Family 282 theorem, whose hypotheses (unitarity, positive energy, discrete spectrum, local net, dilatation current) are not supplied. Contact anomalies are kept distinct from separated-point covariance. The certificate stays RG-open; I-B-VP-5's thm:running named in the plan is absent from the fetched copy (plan correction), so no running statement of I-B-VP-5 is used. par

v5: normalization slots for the general-α product

Definition (Local-covariant diagonal class (LC)). At differential order $\le p$, the ambiguity of the bilocal regular-ball product is a constant-coefficient $O(4)$-covariant symmetric tensor operator applied to $\delta(x-x')$, conserved in x.

Theorem (Slot count under (LC)). *Under (LC) the ambiguities of order* $\le p$ *are exactly* $\sum_{j=0}^{\lfloor (p-2)/2\rfloor} c_j D\Delta^j\delta(x-x')$,

of real dimension 0 *for* $p < 2$ *and* $\lfloor p/2 \rfloor$ *for* $p \geq 2$. *At transverse scaling bound* $s = 4 + p$, $s = 4, \dots, 10$, *the counts are* $0, 0, 1, 1, 2, 2, 3$. *They do not depend on* α.

Proof. An $O(4)$-covariant symmetric symbol is $A(|k|^2)\delta_{\mu\nu} + B(|k|^2)k_\mu k_\nu$ (no parity-odd symmetric tensor from one vector). Conservation $k^\mu(\cdot) = 0$ gives $A + |k|^2 B = 0$, so the symbol is a polynomial multiple of $|k|^2\delta - kk$, the symbol of $-D$; orders $2 + 2j \leq p$ (C5-Q-9). The diagonal of $V \times V$ and the flat symbol calculus do not involve α.

par *Non-claim.* Counts under the declared class (LC) only; Theorem shows the count fails without it. par

Theorem (Without (LC), infinitely many slots at degree six). *For* $f \in C_c^\infty(V)$ *the Hessian kernel* C^f *of* $I_f[A] = \frac{1}{4}\int f F_{\rho\sigma}F^{\rho\sigma}$ *is diagonal-supported, symmetric under slot exchange, conserved in each slot, of scaling degree* ≤ 6, *and depends injectively on f.*

Proof. $(C^f A)_\mu = -\partial_\rho[f(\partial_\rho A_\mu - \partial_\mu A_\rho)]$ is local of order two (support and degree), formally self-adjoint (exchange symmetry), satisfies $\partial_\mu(C^f A)_\mu = 0$ by antisymmetry and $C^f(\partial\xi) = 0$ (both slots). Its principal symbol $f(x)(|k|^2\delta - kk)$ vanishes only if $f = 0$.

par *Non-claim.* The counts concern the bilocal diagonal of codimension four. D6's count $3m^2 + 5m + 3$ (UserDraft6 thm:Q-slotsalpha) is the worldsheet count of Theorem (codimension two), a different object, and is not adopted. The functions f are new coefficient data; physically natural counterterms are not classified. par

v5: scalar Green functions are not the product

Proposition (Running separates scale-independent Green images). *If* G *is a scalar Green operator and* A, B *linear comparison maps, all independent of* v, *then* $v\partial_v(AGB) = 0$, *while* $v\partial_v \Pi^{(v)} = -\frac{1}{24\pi^2}D\delta \neq 0$ *(Theorem). So no such composition equals the family* $\Pi^{(v)}$ *(or* T_α*).*

Proof. Differentiate; equality would equate 0 with the nonzero running.

I-B-ECHO uses the Friedrichs extension on the cone with flux β (I-B-ECHO ass:host), its retarded Green function, and the Carslaw diffracted term (I-B-ECHO eq:Carslaw), which vanishes iff $\alpha^{-1} = N$, $N\beta \in \mathbb{Z}$ (I-B-ECHO

eq:pureimages); I-B-ECHO prop:decomp and I-B-ECHO thm:readout are pulse and readout statements. These are scale-independent scalar objects, so Proposition applies once carrier maps are declared. The VI-DST Green replacement could not be read (404, plan-only locator); the same proposition covers any scale-independent replacement.

par *Non-claim.* An equality at one isolated scale after a new scale-dependent construction is not excluded. A scalar window theorem supplies no vacuum-polarization normalization. par

v5: documentary comparisons

The raw finite part I-B-VP-1 eq:rawfinitepart, the apex product I-B-VP-5 thm:product, the regular-host Ward identities I-B-VP-9 thm:ward ($\nabla\delta T = 0$, $\delta T^{\mu}{}_{\mu} = (6\xi - 1)\mathrm{Box}\, F$) and the resolvent intertwining I-B-VP-CR lem:ward have different carriers and operations; they are comparisons and supply neither the Lorentzian product nor a coefficient. The XVI-O1-CONE-alpha and XVI-O1-CONE-B comparisons (fetched read-only; no labels named in the plan) treat irrational openings with cone-native jets and do not supply the product. XVI-O prop:contacts is named, not used. The OpenAI records OpenAI rem:oai-p-079-01 and OpenAI rem:oai-p-090-04 are documentary.

v5: independent re-verification of v4 claims

1. i ×*EG identification (Theorem , Lemma).* Re-derived by a contour computation: $\int \mathrm{d}q^0/(k^2 - q_0^2 - \mathrm{i}0) = \mathrm{i}\pi/k$ by quadrature at finite $\mathrm{i}0 \to 0$, and the regularized radial transform of $1/|k|$, give $\int \frac{\mathrm{d}^4 q}{(2\pi)^4}\frac{e^{\mathrm{i}qx}}{q^2 - \mathrm{i}0} = \frac{\mathrm{i}}{4\pi^2|x|^2}$ at spacelike x (RV5-Q-1), i.e. i times the Euclidean propagator. Confirmed.

2. *All-α mode sum (Theorem).* The closed form equals the angular series to 40 digits for $\alpha = 0.37, 0.8, 1.6, 2.5$, and the three-image sum at $\alpha = \frac{1}{3}$ (RV5-Q-2). Confirmed.

3. *Naive defect* $(\alpha^{-2} - 1)/(24\pi^2 r^2)$ *(Proposition).* By $\sum_{k<N} \csc^2(\pi k/N) = (N^2 - 1)/3$ (images at distance $2r\sin(\pi k/N)$) and by the limit $h_\alpha \to (\alpha^{-2} - 1)/6$ at non-image α (RV5-Q-3); and by the Lorentzian coincidence value, Theorem (d). Confirmed.

v6: the global structure of the Lorentzian remainder

Notation of Definition . For an off-axis pair put $w = (r^2 + r'^2 + \Delta z^2 - \Delta t^2)/(2rr')$; real Lorentzian pairs have $u = \operatorname{arccosh} w$ real for $w \geq 1$, $u = \mathrm{i}v$ with $v = \arccos w \in [0, \pi]$ for $|w| \leq 1$, and $u = \pm\mathrm{i}\pi + s$, $s > 0$, for $w < -1$ (outside the diffracted front $(r + r')^2 + \Delta z^2 = \Delta t^2$).

Theorem (Singular set, residues and reality of the global continuation). *Fix $\alpha > 0$ and a real θ. Then $u \mapsto h_\alpha(u, \theta)$ is meromorphic on $\mathbb{C}$, and:*

1. *($w > 1$) it has no pole for real $u \neq 0$; at $u = 0$ only if $\theta \in (2\pi\alpha\mathbb{Z} \cup 2\pi\mathbb{Z}) \setminus \{0\}$ (image or developed coincidences).*

2. *($|w| \leq 1$) on $u = \mathrm{i}v$, $0 < v \leq \pi$, its poles are at the image values $v = |\theta + 2\pi\alpha k|$, $k \neq 0$, with residue $1/\sinh u_*$ at each such pole u_* (for $u_k = \mathrm{i}(\theta + 2\pi\alpha k)$: $-\mathrm{i}/\sin(\theta + 2\pi\alpha k)$), and at the diffracted value $v = \pi$, with residue*

$$\operatorname*{Res}_{u = \mathrm{i}\pi} h_\alpha = -\mathrm{i}\,\frac{\sin(\pi/\alpha)}{\alpha(\cos(\pi/\alpha) - \cos(\theta/\alpha))},$$

which vanishes iff $\alpha \in \{1/N : N \in \mathbb{N}\}$ (generic θ; coincident values are excluded).

3. *($w < -1$) the continuations from $|w| < 1$ around the diffracted pole from either side are $h_\alpha(\pm\mathrm{i}\pi + s, \theta)$; they are complex conjugate, and real if $\alpha \in \{1/N\}$; for $\alpha \notin \{1/N\}$ they are not real in general (C6-Q-5). Hence H^L_α continues real-analytically, single-valued and real, to all off-axis Lorentzian pairs with $w > -1$ outside the image cones $\Sigma^{\mathrm{im}}_\alpha = \{w = \cos(\theta + 2\pi\alpha k),\ k \neq 0\}$, with simple poles in w on $\Sigma^{\mathrm{im}}_\alpha$; for $\alpha \notin \{1/N\}$ it has in addition a simple pole on the diffracted front $w = -1$, and its continuation beyond the front is two-valued (complex-conjugate branches), non-real in the computed cases.*

Proof. Poles of f_α: zeros of $\sinh u$ ($u \in \mathrm{i}\pi\mathbb{Z}$) and of $\cosh(u/\alpha) - \cos(\theta/\alpha)$ ($u = \pm\mathrm{i}\theta + 2\pi\mathrm{i}\alpha k$); of f_1: $u = \pm\mathrm{i}\theta + 2\pi\mathrm{i}k$ and none from $\sinh u$, since $f_1 = 1/(\cosh u - \cos\theta)$. (a) For real $u \neq 0$, $\cosh(u/\alpha) > 1 \geq \cos(\theta/\alpha)$ and $\sinh u \neq 0$. (b) At u_k (and likewise at the poles with $-\theta$, by evenness in θ) the numerator $\sinh(u_k/\alpha) = \mathrm{i}\sin(\theta/\alpha)$ cancels against the derivative $\alpha^{-1}\sinh(u_k/\alpha)$ of the bracket, leaving $1/\sinh u_k$; the $k = 0$ poles cancel against f_1 (Theorem (a)), and the poles of f_1 with $k \neq 0$ lie at $v = |\theta + 2\pi k|$, outside $(0, \pi]$ for $|\theta| < \pi$. At $u = \mathrm{i}\pi$, $\sinh u = -(u - \mathrm{i}\pi) + O((u - \mathrm{i}\pi)^3)$ and

$\sinh(\mathrm{i}\pi/\alpha) = \mathrm{i}\sin(\pi/\alpha)$, $\cosh(\mathrm{i}\pi/\alpha) = \cos(\pi/\alpha)$. (c) $h_\alpha(\bar{u},\theta) = \overline{h_\alpha(u,\theta)}$; for $\alpha = 1/N$ all hyperbolic functions at $\pm\mathrm{i}\pi + s$ are real. Near a pole $v_k \in (0,\pi)$, $w - w_k = -\sin v_k\,(v - v_k) + O((v - v_k)^2)$ with $\sin v_k \neq 0$, so poles in v are simple poles in w. Corroboration: C6-Q-1 (diffracted residues; zero at $\alpha = \frac{1}{3}, \frac{1}{2}$), C6-Q-2 (image residues), C6-Q-5 (conjugate branches beyond the front).

par *Non-claim.* A statement about the analytic continuation of the closed-form remainder along real configurations; it constructs no global propagator and no state. The vanishing of the diffracted residue for $\alpha = 1/N$ matches I-B-ECHO eq:pureimages (documentary). par

Corollary (Method obstruction to globalizing the Ward-fixed product).

1. *The Ward condition* $s(x) = 2\mathrm{i}H^L_\alpha(x,x) = \mathrm{i}(\alpha^{-2} - 1)/(24\pi^2 r^2)$ *is global: it holds at every off-axis point, since the diagonal (*$w = 1$, $\theta = 0$*) never meets the singular set.*

2. *The off-diagonal product does not globalize by analytic continuation: on* $\Sigma^{\mathrm{im}}_\alpha$ *(and on* $w = -1$ *for* $\alpha \notin \{1/N\}$*)* H^L_α *has simple poles in w with nonzero residue, so* X_{HH} *(products of first derivatives of H) and* X^F_{cr} *contain non-locally-integrable singularities along these hypersurfaces. The two boundary values* $(w - w_k \pm \mathrm{i}0)^{-1}$ *differ by* $2\pi\mathrm{i}\,\delta(w - w_k)$ *times the residue, which is nonzero. A global product therefore requires a boundary-value prescription on each component of the singular set (state or time-ordering data), which the inputs do not supply and the local construction does not determine.*

3. *For* $\alpha = 1/N$ *the image-sum prescription* $\sigma_k + \mathrm{i}0$ *supplies it; for* $\alpha \notin \{1/N\}$ *the diffracted component needs in addition the global diffracted (Carslaw) term of I-B-ECHO eq:Carslaw, which is not constructed.*

Proof. (a) At $u = \theta = 0$ no pole occurs ((a),(b) of Theorem), and every off-axis point has a regular ball; apply Theorem (e). (b) A simple pole across a hypersurface is not locally integrable after differentiation or squaring, and the Sokhotski–Plemelj formula gives the difference of the boundary values. (c) Theorem (c) and Theorem (b).

par *Non-claim.* This is a method obstruction (continuation does not determine the global product), not a non-existence theorem for a global Lorentzian product with additional data. No time-ordered identification is

made; no quantum coefficient is admitted; the product runs (RG-open). par

v6: the defect class of the actual lift

Definition (Off-diagonal agreement (OD)). A symmetric-tensor-valued distribution L on $V \times V$ (V a regular ball) satisfies (OD) if $L = \Pi^{(v)} - X_{\mathrm{cr}} - X_{HH}$ off the diagonal, with the cone remainder H. The Lorentzian version uses T_v^F, X_{cr}^F, H_α^L.

Theorem (Defect class of every (OD) lift). *Every lift L with (OD) is, locally, $L = L_{H,0,0} + \sum_{|\gamma| \le k} C_{\mu\nu}^{\gamma}(x)\,\partial^{\gamma}\delta(x - x')$ with C^{γ} distributions on V, and*

$$\partial_x^{\mu} L_{\mu\nu} = 2H(x', x')\,\partial_{\nu}\delta(x - x') + \partial_x^{\mu}\Big(\sum_{\gamma} C_{\mu\nu}^{\gamma}\partial^{\gamma}\delta\Big)$$

(Lorentzian: $-2\mathrm{i}H_\alpha^L(x', x')\partial_\nu\delta + \dots$). It is removed by $R_L = -\sum_{\gamma} C^{\gamma}\partial^{\gamma}\delta - 2H(x,x)\delta_{\mu\nu}\delta$ (Lorentzian: $+2\mathrm{i}H_\alpha^L\eta_{\mu\nu}\delta$), and the set of all diagonal-supported removals is R_L plus the conserved diagonal-supported tensors. Consequently:

1. *in the quotient of diagonal-supported defects by divergences of diagonal-supported symmetric corrections, the defect class of every (OD) lift is zero, so the support separator (Proposition) detects no (OD) lift;*

2. *the defect itself is not determined by (OD): every element of*

$$2H(x', x')\partial_{\nu}\delta + \mathrm{div}\,(\text{diagonal-supported symmetric})$$

is the defect of some (OD) lift;

3. *under (LC) and an order bound p, the removal is unique up to the $\lfloor p/2 \rfloor$ slots of Theorem .*

Proof. $L - L_{H,0,0}$ is supported on the diagonal, so by the structure theorem for distributions supported on a submanifold it is locally a finite sum $\sum C^{\gamma}\partial^{\gamma}\delta(x - x')$ in the coordinates $(x, x - x')$. The divergence of $L_{H,0,0}$ is $2H(x', x')\partial_\nu\delta$ (Theorem with $a = 0$, $C = 0$; Lorentzian Theorem (e)). $L + R_L = L_{H,-2H,0}$ is conserved. Two removals differ by a conserved diagonal-supported tensor. (b) Take $L = L_{H,0,0} + B$. (c) Theorem .

par *Non-claim.* (OD) is a declared hypothesis on the actual inherited lift, which is not constructed; the actual

coefficients C^{γ} are not computed and are not determined by the inputs. The statement tightens Theorem from a class of smooth-remainder lifts to every lift with the cone's off-diagonal values. No quantum coefficient is admitted; RG-open. par

v6: documentary comparisons

VI-DST now resolves read-only (sha256 45042eed…). VI-DST thm:replacement (exact scalar Green-replacement sensitivity) and VI-DST thm:window (survival in a complete transfer packet) are scale-independent scalar statements, so Proposition applies to them once carrier maps are declared; they supply no vacuum-polarization normalization. The diffracted residue condition of Theorem coincides with the pure-image condition of I-B-ECHO eq:pureimages; that is a comparison, not a premise.

v6: independent re-verification of v5 claims

1. *Coincidence value* $(\alpha^{-2}-1)/(48\pi^2 r^2)$ *(Theorem (d)).* v5 used series expansion. v6 recomputes $h_\alpha(0,0) = (\alpha^{-2}-1)/6$ as numerical limits along five rays (spacelike, timelike $u = \mathrm{i}v$, mixed) at 30 digits for $\alpha = 0.37, 0.8, 2.5$ (C6-Q-3). Confirmed.

2. *Ward coefficient* $s = 2\mathrm{i}H$ *(Theorem (e)).* v5 used the Leibniz computation. v6 checks the same algebra by a direct distributional pairing in the one-dimensional analogue $G = \frac{\mathrm{i}}{2}|x|$ ($G'' = \mathrm{i}\delta$) with a symmetric bisolution H, by two-dimensional quadrature (C6-Q-4): the pairing of $\partial_x X_{\mathrm{cr}}$ equals that of $2\mathrm{i}H(x', x')\partial_x \delta$. The identity uses only $\mathrm{Box}\, G = \mathrm{i}\delta$, $\mathrm{Box}\, H = 0$ and symmetry, so the analogue tests exactly the step used. Confirmed.

Release

Theorem (I-B-G-Q-B release).

1. *One renormalized interacting product* $\Pi^{(v)}$ *of a declared interaction is constructed, with descent, an exact Ward identity, covariance, and uniqueness up to the scale (Theorem).*

2. *For it, descent, Ward and normalization hold together; scale independence fails (Theorem).*

3. *It is not the inherited coefficient (Theorem).*

4. *Under the typing (T0)–(T3), the inherited coefficient is not determined by the inputs, hence not admitted (Theorem).*

5. *The typing alone decides neither success nor failure of the actual-image condition (Theorem). A failure would follow from a nonzero defect of the actual lift off the correction supports (Proposition); that defect is not evaluated.*

6. *(v2) The running of* $\Pi^{(v)}$ *is exact in position and momentum space (Theorem).*

7. *(v2) On regular balls of the Euclidean section of the prescribed cone with* $\alpha = 1/N$*, a product with exact Ward identity is constructed; the seagull term is forced; uniqueness holds up to the scale under local covariance (LC), and fails without it (Theorem , Proposition). The Lorentzian transfer lacks exactly one flat datum; other deficits are outside the method (Theorem).*

8. *(v2) The translation-invariant admissible family has dimension* $25, 45, 71$ *for* $\omega = 4, 5, 6$ *(*$3, 3, 5$ *covariant), and that many real data are necessary and sufficient (Theorem).*

9. *(v3) The Euclidean product and its proved properties do not determine a flat Lorentzian product, even modulo local counterterms (Theorem); one explicit declared datum with Ward identity and exact running is written down, its identification with the time-ordered product not being proved (Proposition).*

10. *(v3) Finite images realize the cone exactly for* $\alpha = 1/N$ *(Theorem). The actual-lift detecting value and its sign are not determined by the typed inputs (Proposition). In every degree the* $d_m = 3m^2 + 5m + 3$ *normalization slots (*$2\lfloor m/2 \rfloor + 1$ *covariant) are unfilled by every subset of the typed inputs (Theorem , Corollary). Hypothesis (T0) is corrected to NSS.*

11. *(v4) The declared Feynman datum is* i *times the unitary Epstein–Glaser time-ordered product, unique up to v; unitarity is exactly the extra datum (Theorem). The Euclidean cone product exists for every* $\alpha > 0$ *(Theorem). On*

the declared model the naive-lift defect is $(\alpha^{-2} - 1)/(24\pi^2 r^2)\,\partial\delta$*, diagonal and removable (Proposition).*

12. *(v5) The Lorentzian local cone remainder exists for every* $\alpha > 0$ *and fixes the seagull* $2\mathrm{i}H^L_\alpha(x,x)$*; no state or time-ordered identification (Theorem). A class of lifts has a removable local defect (Theorem); the running does not exclude the Oai282-A Ward certificate (Theorem); slots are counted under (LC) and infinite without (Theorems ,); scale-independent Green functions are not the product (Proposition).*

13. *(v6) Globalizing the Lorentzian cone remainder meets a method obstruction (a boundary-value prescription) off the diagonal, where* $s = 2\mathrm{i}H$ *is global (Theorem , Corollary); the defect class of every (OD) lift is zero, its representative undetermined (Theorem). RG-open stays.*

Proof. Collect the results of this sheet.

par *Non-claim.* No bulk BV restoration, no O.1 closure, no ultraviolet completion, and no spacetime emergence are claimed. par

Release box (I-B-G-Q-B v6). Interacting Ward identity: *recorded for one product* (constructed). Scale independence for that product: *fails*. Inherited coefficient: *separated under the stated typing* (non-determination proved); not admitted. Failure of the actual-image condition: not proved (conditional separator only). v2: exact running $-\frac{1}{24\pi^2}\log(v'/v)$; cone product for $\alpha = 1/N$ (Euclidean, regular balls) with Ward identity; Lorentzian transfer conditional on one flat datum; free family of dimension 25 at $\omega = 4$ (covariant: 3). v3: flat Lorentzian product *not determined* by the Euclidean data (two-parameter non-local family); one explicit declared datum, identification with the time-ordered product not proved; finite images iff $\alpha = 1/N$; actual-lift value and sign *not determined* by the typing, not evaluated; all $3m^2 + 5m + 3$ slots unfilled; (T0) corrected to NSS. v4: declared Feynman datum identified with i × the unitary EG product (flat datum supplied); Euclidean cone product for every α; declared-model defect evaluated (local, removable). Lorentzian cone product for general α not constructed; actual-lift defect not evaluated. v5: Lorentzian local cone remainder for every α (no state, no TO identification); defect of a class of lifts; running compatible with the Oai282-A certificate; slots counted. Actual-lift defect not evaluated; product runs (RG-open). v6: globalization obstructed off the diagonal (boundary-value prescription);

actual-lift defect class zero, representative undetermined. Product runs (RG-open). No quantum coefficient is admitted. The certificate remains RG-open. Bulk BV restoration is not claimed. O.1 stays NOT_CLOSED. Host C is not created. XVI-N is not discharged. XV-B is not opened. Cited sheets are documentary, never premises. No compilation is asserted. No independent review has been performed.

Relation to existing sheets

This sheet is new; it revises nothing. I-B-G-Q supplies the criterion (thm:compatibility), the finite part and running warning (thm:finitepart, prop:conserved-running), the three typing propositions and the negative release (thm:release). They are restated in Section and are not premises: the finite part is re-proved (Lemma). The I-B-VP finite parts and the XVI-O kernel counterterm are named and set aside (Section). Bulk BV restoration belongs to XVI-O-B and is not addressed.

Separation and edge table

toprule Sheet Labels Edge (what is used, what is separated)

midrule endhead I-B-G-Q thm:compatibility, thm:finitepart, prop:conserved-running, prop:v5-codim2, prop:v5-ward, prop:v5-global, thm:release Criterion restated and applied to one product; typing used to state (T1)–(T3); release sharpened under the stated typing (non-determination), and kept RG-open.

I-B-VP-1 sec:finiteparts, eq:rawfinitepart Finite parts of the I-B-VP series. Named; not used, since the product's finite part is re-proved on $\mathbb{R}^4$. Separated.

XVI-O prop:kappa2-master, prop:contacts, lem:analytic_FP_u_mu_R_o70 The kernel counterterm lives on S_{inv} and is not this product. Not inserted. Separated.

I-B-G thm:green Ledger Green operators are not an interacting product. Separated.

I-B-G prop:nonGH The prescribed cone is not globally hyperbolic; general Hadamard uniqueness is not invoked (Theorem). Documentary.

OpenAI rem:oai-p-090-04, rem:oai-p-079-01 Catalogue records; documentary only.

Oai282-A ass:ward, thm:ward (v5) Formal transcription (Theorem).

I-B-ECHO ass:host, eq:Carslaw, eq:pureimages, prop:decomp, thm:readout (v5) Comparison (Proposition).

I-B-VP-5 thm:product (v5) Documentary.

I-B-VP-9 thm:ward (v5) Documentary.

I-B-VP-CR lem:ward (v5) Documentary.

I-B-VP-1 eq:rawfinitepart (v5) Documentary.

I-B-ECHO eq:Carslaw, eq:pureimages (v6) Diffracted and image terms (Theorem).

VI-DST thm:replacement, thm:window (v6) Documentary (Section).

bottomrule

Open table

toprule Open item Status after this sheet

midrule endhead Inherited coefficient Separated under the stated typing (non-determination); not admitted.

Failure of the actual-image condition Not proved; conditional on a nonzero defect of the actual lift.

Interacting Ward identity Recorded for one declared product only.

Certificate RG-open.

Bulk BV restoration Not claimed (XVI-O-B).

O.1 NOT_CLOSED.

Cone product, general α Not supplied; image method limited to $\alpha = 1/N$ (Theorem (b)).

Lorentzian cone product Conditional on the flat Lorentzian renormalized product (Theorem (a)).

Normalization data for the inherited coefficient Exactly $\dim K_{\omega}$ real numbers under invariance (Theorem); not

supplied.

Identification of the declared Lorentzian datum with the time-ordered product Not proved (Proposition); Theorem (a) stays conditional.

Products for $\alpha \notin \{1/N\}$ Method obstruction only (Theorem); existence not decided.

Actual-lift defect Not evaluated; not determined by typing (Proposition).

Normalization data d_m numbers missing in degree m (Corollary).

Lorentzian cone product, $\alpha \notin \{1/N\}$ (v4) Not constructed (no positive two-point function with remainder H_α supplied).

Lorentzian cone Ward identity with the EG datum (v4) Not re-proved.

Actual-lift defect (v4) Still not evaluated; only a declared-model lift is.

Global Lorentzian cone product and state, $\alpha \notin \{1/N\}$ (v5) Not constructed; local remainder only (Theorem).

Actual inherited lift and its defect (v5) Not constructed; class of lifts evaluated.

Boundary-value prescription off the diagonal (v6) Not supplied by the inputs; method obstruction (Corollary).

Representative of the actual-lift defect (v6) Undetermined; class zero for every (OD) lift.

bottomrule

appendix

Plan corrections

1. The plan cites an ``I-B-VP finite-part label''. The main I-B-VP sheet has none. The finite parts are in sec:finiteparts and eq:rawfinitepart of I-B-VP-1, which are cited instead.

2. The plan asks for a product ``on the prescribed conical setting''. v1 constructed it on auxiliary Euclidean $\mathbb{R}^4$ and did not claim a conical product. (v2) A conical product is now constructed on regular balls of the Euclidean

section for $\alpha = 1/N$ (Theorem); the Lorentzian version needs one flat datum, and other deficits are outside the method (Theorem). The obstruction branch is unaffected.

3. The plan's ``cannot be admitted'' is proved in the form ``is not determined by the typed inputs'' (Theorem). Failure of the actual-image condition is not proved and, by Theorem , cannot follow from the typing alone.

4. The I-B-G-Q site URL returned HTTP 404. The local v5 copy was read and all labels were verified there. The site URL is a plan-only locator.

5. (v3) The plan's item (1) offers construction or non-determination. The non-determination branch is proved. A declared datum is written, but constructing ``the'' Lorentzian product would need the time-ordering identification, which is not proved here, so the construction branch is not claimed.

6. (v3) I-B-G-Q, formerly 404 at the site, now serves and is byte-identical to the local copy. All plan labels for this sheet (thm:compatibility, thm:finitepart, prop:conserved-running, prop:v5-codim2, prop:v5-ward, prop:v5-global, I-B-VP-1 thm:U, def:Ward, eq:rawfinitepart, I-B-G thm:green, prop:nonGH, I-B-G-S thm:spin1, XVI-O prop:contacts) were found.

7. (v3) OpenAI items 090-04 and 079-01 were located as rem:oai-p-090-04, rem:oai-p-079-01; cited as catalogue records only.

8. (v4) ``Identify the declared Feynman datum with a time-ordered product'': it is i times that product, and uniqueness needs unitarity, which is an EG axiom beyond the typed inputs.

9. (v4) ``Give the sign/size of the actual-lift defect on a declared model'': the actual lift is not constructed; the sign and size are given for the naive lift of the declared free-field model, which is a different object.

10. (v5) Item (1): the alternative ``construct by mode sum or method obstruction'' is resolved by a third method (analytic continuation), locally; D6's all-α method obstruction is not proved and not adopted.

11. (v5) Item (3): ``the running places the product outside thm:ward/Family 282'' is false as stated (Theorem).

12. (v5) I-B-VP-5 thm:running is absent from the fetched copy (only thm:product found); VI-DST

thm:replacement returned 404 (plan-only locator); XVI-O1-CONE-alpha/-B were fetched but no labels were named, so none is cited.

13. (v6) No new plan or draft was supplied for v6; the targets are the v5 open items and the items v5 only narrowed.

14. (v6) VI-DST now resolves (HTTP 200 on 2026-10-22, local read-only copy v6/ inputs/ src/ VI-DST.api.tex); its plan-only locator status is lifted for VI-DST itself. VI-DST-A and VI-DST-v1 still return 404 and are not cited.

Errors

1. No v5 statement was found to be false in v6 (re-verified by C6-Q-3, C6-Q-4).

2. Status update, not an error: the v5 VI-DST 404 note is superseded (VI-DST resolves on 2026-10-22).

Errors recorded in v5 (retained)

1. No v4 statement was found to be false in v5 (re-verified by RV5-Q-1 to RV5-Q-3).

2. Status update, not an error: the v4 open item on the Lorentzian product for $\alpha \notin \{1/N\}$ is narrowed by Theorem .

Errors recorded in v4 (retained)

1. Proposition , non-claim (v3). It said $\mathfrak{P}_v^F$ ``restricts … to the time-ordered pointwise product". The correct statement is $\mathfrak{P}_v^F = \mathrm{i}\, T^\circ$ off the origin (Lemma). The proposition is unaffected.

Errors recorded in v3 (retained)

1. (T0) was false for the benchmark (v2, Section). v2 required every point of W to lie in the closure of $\operatorname{sing\,supp} u^\circ$ and asserted that $|x_\perp|^{-2} \otimes 1$ satisfies it. That function is smooth on $\mathbb{R}^4 \setminus W$, so its singular support there is empty and the hypothesis fails. Corrected: (T0) is now NSS (no smooth local extension at any point of W), which the benchmark satisfies (C-Q-23), and the proof of Theorem is rewritten (smoothness of an extension near w would give a smooth local extension). The conclusion is unchanged. Found by the user draft D5 (item E1); verified.

2. ``Applies verbatim'' (v2, non-claim of Theorem). Theorem compares distributions on open subsets of $\mathbb{R}^4$; the cone product is bilocal on $V \times V$. The separation is by type, not by verbatim application. Corrected in the text. (Draft D5; verified.)

3. No error was found in Theorem on re-verification.

Errors recorded in v2 (retained)

1. Imprecise count (corrected). v1, proof of Theorem (b): ``This is consistent with prop:v5-codim2(ii): with invariance, one real number remains free." That proposition concerns a scalar datum at minimal degree $\omega' < 3$. For the symmetric conserved tensor class at $\omega \geq 4$, translation invariance leaves 25 (covariantly 3) real numbers free (Theorem). The theorem's statement (``contains the one-parameter family") was and is correct; only the gloss understated the freedom.

2. Plan correction withdrawn. v1 plan correction 2 is superseded (see Changes). It was a correct statement of what v1 supplied, not a false mathematical claim.

3. No error was found in the v1 Ward identity, running coefficient, dilation formula, or uniqueness proof (Section).

Relation to the user drafts

Seven user drafts have been supplied, all copied read-only: **D1** *Five Separation Sheets* (inputs/UserDraft-FiveSeparationSheets.tex, sha256 b9f18ae9…); **D2** *Five Separation Developments* (inputs/UserDraft2-FiveSeparationDevelopments.tex, 6f596e42…); **D3** *Five Separation Sheets Beyond Version 5* (inputs/UserDraft3-FiveSheetsBeyondV5.tex, c9636381…); and, for v3, **D4** (draft A) *Five Separation Sheets, Version 3 — Closing named open items without closing standing refusals* (v3/inputs/draftA_c9cbf1eb.tex, c9cbf1eb…) and **D5** (draft B) *Five separation sheets, version 3 — consolidated development and scoped closure dossier* (v3/inputs/draftB_e98f0a94.tex, e98f0a94…); and, for v5, **D6** *Version 5 — Closing named open items* (v5/inputs/UserDraft6-V5Closing.tex, f7b5578f…) and **D7** *Five separation sheets, version 5 — consolidated*

development and correction dossier (v5/inputs/UserDraft7-Dossier.tex, 45ef0568…). They are treated as the user's outline: each statement used was re-verified (mathematics, labels, scope) and is re-proved in this sheet; nothing is taken as a premise. The originals are not edited. Full hashes are in the artifact table. No new draft was supplied for v6; the reconciliation below is retained and unchanged.

toprule Draft Item Decision Reason

midrule endhead D1 def:product Instantiated D1 leaves the product unspecified (``supply a typed renormalized product''). Instantiated by Definition .

D1 thm:gauge Corrected D1 asserts all four conditions, including scale independence, ``by the declared typing''. False for the constructed product: scale independence fails (Theorem (ii)). An admission of non-running would be refused in any case.

D1 thm:nonadm Re-proved D1's proof is an assertion. Replaced by the singular-support argument of Theorem .

D1 thm:obstr-coeff Re-proved, scoped D1's proof says only that uniqueness ``is not supplied''. Replaced by Theorem , with an explicit injective conserved family and the monodromy clause.

D2 thm:Q-actual-image Adopted Restatement of thm:compatibility; used in Section .

D2 lem:Q-transport Correct; not used A valid algebraic lemma (transport of a Ward product by an invertible R commuting with q). It constructs no analytic product, as D2 itself notes.

D2 thm:Q-typing-insufficient Adopted, re-proved Theorem . Checked against the watch item ``limit of a typing obstruction'': it limits only *failure* claims. Theorem is a non-determination claim and is not weakened.

D2 prop:Q-separator Adopted, specialized Proposition (support form). D2's caveat that the separator is not evaluated is kept.

D2 prop:Q-running Consistent Same warning as Proposition .

D3 def:Q-product-typing, lem:Q-div-identity, thm:Q-product-ward Not used A conformal-scalar stress two-point product on a conical wedge. The divergence lemma's proof changes its coefficient mid-proof (``Hold on: …''); it

uses I-B-VP and XVI-O kernels as premises; its wavefront claims are not proved. The constructed product here is independent of it.

D3 prop:Q-not-inherited Idea acknowledged The graph and valence separation is plausible and consistent with Theorem , but it relies on the unadopted D3 product. The singular-support argument is used instead.

D3 thm:Q-obstruction Corrected (downgraded) Part (i), that corrections are supported on diagonals and the worldsheet, is accepted as the typing. Part (ii) is correct only conditionally: it needs the defect of the *actual* lift to be nonzero on U_{sep}. D3 supplies a degree-filtered probe and the XIV-K-ST ambiguity A_N instead, which are not that defect. ``Impossibility'' in (iii) is refused. What remains is recorded as Proposition .

D3 prop:Q-running, thm:Q-release Consistent / adopted in part Release adopted without ``impossibility''.

bottomrule *Reconciliation.* The three drafts disagree on the strength of the obstruction. D1 asserts it, D2 shows that typing cannot prove failure, and D3 claims failure. These are reconciled as follows. Non-determination is proved (this sheet). Failure would follow from a nonzero defect of the actual lift off the correction supports (D2/D3 combined, Proposition). That defect is not evaluated, so D3's impossibility claim is not recorded. On the product branch, D1's ``all four conditions'' is wrong, D2 constructs none, and D3's construction is not verified; this sheet's flat Euclidean product is the one recorded.

v2 update. No draft constructs a conical product; v2 supplies one for $\alpha = 1/N$ (Theorem), consistent with D2's typing-insufficiency theorem (the product does not touch the actual-image condition of the inherited coefficient). The watch item ``limit of a typing obstruction implying a weaker I-B-G-Q-B obstruction'' was re-checked: Theorem strengthens the non-determination (exact dimension), and nothing in v2 weakens Theorem . D3's failure witnesses remain not adopted.

v3 update (drafts D4 = A and D5 = B).

toprule Draft Item Treatment

midrule endhead D4 Q-flat (Lorentzian product) Asserted by Wick rotation without proof. **Refused as a**

construction; replaced by Theorem and the declared datum of Proposition .

D4 general α, slots Conclusions correct, proofs absent. **Proved** here (Theorems ,).

D5 def:Q-nonremovable, thm:Q-correctedseparator **Adopted after verification** as the corrected (T0) (error item 1).

D5 lem:Q-log, thm:Q-nontransfer **Adopted**, extended to a two-parameter family with non-local differences.

D5 thm:Q-imageboundary **Adopted**, with the isometry-invariant argument.

D5 prop:Q-defectgap **Adopted**.

D5 thm:Q-allcount, cor:Q-unfilled **Adopted**; the transverse block derivation and the covariant count are written out, and the formula recomputed (C-Q-18, C-Q-19).

D1–D5 (v4) **Unchanged**. No draft item is re-decided in v4; D5's Q-nontransfer is complemented by the EG identification of Theorem .

D6 (v5) **Corrected.** thm:Q-lorentzalpha (method obstruction for every α) not adopted: the local product is constructed (Theorem); thm:Q-defectclass restricted to the class of Theorem ; thm:Q-runsep refuted (Theorem); thm:Q-slotsalpha ($3m^2 + 5m + 3$) is the worldsheet count, not the cone diagonal (Theorem); thm:Q-green adopted via Proposition .

D7 (v5) **Adopted after re-verification.** thm:Q-Lmethod (as a lemma), thm:Q-liftclass, thm:Q-conecount, thm:Q-infslots, thm:Q-conformal, prop:Q-Gseparation (C5-Q-7 to C5-Q-9); D7's statement that the Oai282-A Ward theorem was not supplied is superseded (Oai282-A thm:ward was read).

bottomrule

Label audit

Every label cited with the source-locator macro in this sheet was checked by the build script against the input file actually read (copied read-only into inputs/). A missing label stops the build. toprule Sheet Label Input file Origin

Result

midrule endhead I-B-G-Q thm:compatibility I-B-G-Q-v5.tex local v5 = site (2026-10-22) found

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XVI-O prop:contacts XVI-O.tex site fetch 2026-10-22 found

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VI-DST thm:window ../v6/inputs/src/VI-DST.api.tex site fetch 2026-10-22 (v6; resolves now) found

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bottomrule

Computer-assisted checks

The following checks were run by the scripts named in the artifact table. Symbolic checks are exact identities in sympy; interval checks use outward-rounded mpmath.iv arithmetic. Checks corroborate the hand proofs; no

theorem rests on a check unless the theorem says so. toprule Id Statement Result

midrule endhead C-Q-1 $2\partial_\mu \Delta\, \partial_\nu \Delta - 2\Delta\, \partial_\mu \partial_\nu \Delta = c_0(\partial_\mu \partial_\nu - \delta_{\mu\nu}\Delta)|x|^{-4}$ off 0, $c_0 = -1/(48\pi^4)$, exact. pass

C-Q-2 $(\partial_\mu \partial_\nu - \delta_{\mu\nu}\Delta)|x|^{-4} = -12|x|^{-6}(\delta_{\mu\nu} - 2x_\mu x_\nu/|x|^2)$, exact. pass

C-Q-3 Covariant point-supported symmetric terms of order ≤ 2: the Ward identity holds iff $\alpha = 0$, $\beta = -\gamma$, exact solve. pass

C-Q-4 The symbol of $\partial_\mu \partial_\nu - \delta_{\mu\nu}\Delta$ is transverse, exact. pass

C-Q-5 $u_\nu(\varphi(\cdot/\lambda)) = u_{\lambda\nu}(\varphi)$ for a radial Gaussian, $\lambda = 3$, $\nu = 2$ (numerical, 30 digits; corroboration). pass

C-Q-6 Trace of the running tensor symbol is -3Δ, exact. pass

C-Q-7 The image term $G_0(x - R_\theta x')$ is harmonic in x, and $G_0(x - R_\theta x') = G_0(x' - R_{-\theta}x)$, exact. pass

C-Q-8 $|x - R^k x|^2 = 4r^2 \sin^2(\pi k/N)$, exact; $\sum_{k=1}^{N-1} \csc^2(\pi k/N) = (N^2 - 1)/3$ for $N = 2, \ldots, 12$ (40 digits, rational recognition). pass

C-Q-9 Divergence of the cross terms equals $2\Delta G_0\, \partial' H + 2\Delta H\, \partial' G_0 - 2G_0 \partial' \Delta H - 2H\partial' \Delta G_0$ for generic smooth functions (pointwise Leibniz algebra), exact. pass

C-Q-10 Divergence of the HH terms equals $2\Delta H\, \partial' H - 2H\, \partial' \Delta H$, exact. pass

C-Q-11 For symmetric $H(a, b) = F(a, b) + F(b, a)$ with an explicit nonsymmetric F: $(\partial_1 H)(p, p) = (\partial_2 H)(p, p)$, exact. pass

C-Q-12 $\pi^2 4^\varepsilon \Gamma(\varepsilon)/\Gamma(2 - \varepsilon)\, K^{-2\varepsilon}$ has $\log K$ coefficient $-2\pi^2$ at order ε^0; $\pi^2|c_0| = 1/(48\pi^2)$; $2\pi^2 c_0 = -1/(24\pi^2)$, exact. pass

C-Q-13 Fourier formula in $\mathbb{R}^4$ corroborated numerically through the Hankel integral at $a = 3.5$ (20 digits). pass

C-Q-14 Off-diagonal identity of Lemma at $x = (0.3, -0.7, 0.45, 1.1)$ by central finite differences (40 digits), a method independent of C-Q-1. pass

C-Q-15 Dimension of translation-invariant, conserved, W-supported symmetric ambiguities with $\mathrm{sd}_W \le \omega$: $25, 45, 71$ for $\omega = 4, 5, 6$; with $SO(1,1) \times SO(2)$ covariance $3, 3, 5$ (exact rank computation); v2/logs/q_dims.json. pass

C-Q-17 The three covariant generators at $\omega = 4$ are conserved, exact. pass

C-Q-16 The symbol of $\Theta(1)$ is conserved, exact. pass

C-Q-18 $\dim K$ at normal order $\le m$, computed by floating-point SVD of the conservation system (a different method from the exact rank of v2, C-Q-15), equals $3m^2 + 5m + 3$ for $m = 0, \ldots, 7$; $m = 2, 3, 4$ give $25, 45, 71$. pass

C-Q-19 $2\lfloor m/2 \rfloor + 1$ equals $3, 3, 5$ at $m = 2, 3, 4$ (covariant count of thm:Q-dim). pass

C-Q-20 $a[\log(q + \mathrm{i}0) - \log(q - \mathrm{i}0)] = \mathrm{i}/(24\pi)$ on $q < 0$, $a = 1/(48\pi^2)$, exact. pass

C-Q-21 Lorentzian $P_{ab} = \eta_{ab} q - k_a k_b$ is transverse, and $v\partial_v$ of $a[\log(|q|/v^2) + \mathrm{i}\pi\theta(-q)(\alpha + \beta \operatorname{sgn} k_0) - c]$ is $-1/(24\pi^2)$ independently of α, β, exact. pass

C-Q-22 For $\gcd(p, q) = 1$, $q \le 12$, rotation by $2\pi p/q$ generates $\mathbb{Z}_q$ with smallest positive angle $2\pi/q$ (enumeration). pass

C-Q-23 $|x_\perp|^{-2} \to \infty$ as $x_\perp \to 0$: no continuous, hence no smooth, local extension; it is smooth off W. pass

C4-Q-1 Schwinger constant $C(\lambda)$: $C(-1) = \mathrm{i}/(4\pi^2)$ and $C(\lambda)/\lambda \to -\mathrm{i}/\pi^2$ (symbolic). pass

C4-Q-2 $(\partial\partial - \eta\mathrm{Box})(x^2)^{-2} = 24\, xx(x^2)^{-4} - 12\eta(x^2)^{-3}$ and $2(\partial\Delta\partial\Delta - \Delta\partial\partial\Delta) = c_0(\partial\partial - \eta\mathrm{Box})(x^2)^{-2}$, $c_0 = -1/(48\pi^4)$ (symbolic, $(-+++)$). pass

C4-Q-3 $-\mathrm{i}a \cdot (-\mathrm{i}/\pi^2) = -a/\pi^2 = c_0$ (exact). pass

C4-Q-4 Plemelj jump $\log(q - \mathrm{i}0) - \log(q + \mathrm{i}0) = -2\pi\mathrm{i}$ at $q < 0$; difference of modulus $2\pi a = 1/(24\pi)$. pass

C4-Q-5 Local covariant symmetric terms $A\eta + Bkk + C\eta q$ are transverse iff $A = 0$, $B = -C$ (symbolic solve). pass

C4-Q-6 Cone mode identities: $|\nabla u|^2 = r^{-2}$, $\Delta A = 0$ (exact); $2\nabla A \cdot \nabla u + A\Delta u = 0$ (50 digits, 6 points). pass

C4-Q-7 $f_\alpha - f_1 \to (1-\alpha^2)/(6\alpha^2)$ in every direction (series in polar coordinates, symbolic). pass

C4-Q-8 For $\alpha = 1/N$, $N = 2, 3, 5$: closed form G_α equals the image sum (30 digits, 9 point pairs). pass

C4-Q-9 $\Delta_x G_\alpha = 0$ off the diagonal for $\alpha = 0.37, 0.8, 1.6$ (40-digit numerical differentiation). pass

C4-Q-10 Defect coefficient $2H_\alpha(x,x) = (\alpha^{-2}-1)/(24\pi^2 r^2)$; value $1/(8\pi^2)$ at $\alpha = \frac{1}{2}$, $r = 1$; equals $(N^2-1)/(24\pi^2 r^2)$ at $\alpha = 1/N$. pass

RV4-Q-1 $d_m = 3m^2 + 5m + 3$ by exact rational rank, $m \le 9$; generating function $(3 + 2t + t^2)/(1-t)^3$. pass

C5-Q-1 Residue of f_α along $u = \mathrm{i}\theta$ equals $1/\sinh(\mathrm{i}\theta)$ for $\alpha = 0.37, 0.8, 1.6, 1$ (60 digits): independent of α. pass

C5-Q-2 H^L_α is real and finite on both sides of the developed light cone; one-sided extrapolations agree there. pass

C5-Q-3 $\mathrm{Box}_x H^L_\alpha = 0$ at a timelike- and a spacelike-separated pair, $\alpha = 0.37, 0.8, 1.6$ (60-digit numerical differentiation). pass

C5-Q-4 For $\alpha = 1/N$ ($N = 2, 3, 5$), H^L_α equals the Lorentzian image sum $\sum_k 1/(4\pi^2\sigma_k)$ at a timelike pair. pass

C5-Q-5 Coincidence limit of H^L_α along a timelike direction is $(\alpha^{-2}-1)/(48\pi^2 r^2)$, $\alpha = 0.37, 0.8, 1.6$. pass

C5-Q-6 With $\mathscr{F}^{-1}[(q - \mathrm{i}0)^{-1}] = \mathrm{i}\Delta_F$ and Box $\to -q$: $\mathrm{Box}\,\Delta_F = \mathrm{i}\delta$ (exact). pass

C5-Q-7 $\langle u_v, -x\cdot\partial\varphi\rangle = 2\pi^2\varphi(0)$; $[x\cdot\partial, D] = -2D$; $\mathrm{tr}\,D = -3\Delta$; hence $(x\cdot\partial + 6)\Pi_v = -\frac{1}{24\pi^2}D\delta$ and $\Pi_v = -\frac{1}{3}D(L)$ (symbolic). pass

C5-Q-8 $D_{\mu\nu}|x|^{-4} = -12|x|^{-6}(\delta - 2xx/|x|^2)$ and conformal covariance of Π° under inversion at 20 random pairs. pass

C5-Q-9 $O(4)$-covariant conserved diagonal symbols $A(t)\delta + B(t)kk$ of order $\le p$: dimension $\lfloor p/2 \rfloor$ for $p \ge 2$, 0 otherwise, $p \le 10$ (exact). pass

RV5-Q-1 i ×EG by contour quadrature: $\int \mathrm{d}q^0/(k^2 - q_0^2 - \mathrm{i}0) = \mathrm{i}\pi/k$ and the regularized radial transform of $1/|k|$ give $\mathrm{i}/(4\pi^2|x|^2)$ at a spacelike point. pass

RV5-Q-2 All-α Euclidean kernel: closed form equals the angular mode series ($\alpha = 0.37, 0.8, 1.6, 2.5$, 40 digits);

three-image sum at $\alpha = \frac{1}{3}$. pass

RV5-Q-3 Naive defect by $\sum_{k<N} \csc^2(\pi k/N) = (N^2 - 1)/3$ $(N \le 8)$ and by $h_\alpha \to (\alpha^{-2} - 1)/6$ for non-image α. pass

C6-Q-1 Diffracted residue $\mathrm{Res}_{u = \mathrm{i}\pi} h_\alpha = -\mathrm{i}\sin(\pi/\alpha)/(\alpha(\cos(\pi/\alpha) - \cos(\theta/\alpha)))$ by contour quadrature; zero at $\alpha = \frac{1}{3}, \frac{1}{2}$. pass

C6-Q-2 Image residues $1/\sinh u_k$ at $u_k = \mathrm{i}(\theta + 2\pi\alpha k)$, $k = \pm 1$, $\alpha = 0.37, 0.45$. pass

C6-Q-3 Re-verification of $h_\alpha(0,0) = (\alpha^{-2} - 1)/6$ by numerical limits along five rays (incl. timelike), $\alpha = 0.37, 0.8, 2.5$. pass

C6-Q-4 Re-verification of $s = 2\mathrm{i}H$: one-dimensional analogue, distributional pairing of $\partial_x X_{\mathrm{cr}}$ by quadrature equals that of $2\mathrm{i}H(x', x')\partial_x \delta$. pass

C6-Q-5 Beyond the diffracted front ($u = \pm\mathrm{i}\pi + s$) the two continuations are complex conjugate; real for $\alpha = \frac{1}{3}, \frac{1}{2}$, not for $\alpha = 0.37, 0.8$. pass

bottomrule

Artifacts and sha256

Paths are relative to /workspace/NewSheets. The hash of this sheet's own source and PDF is recorded in the external manifest MANIFEST-NewSheets-v6.sha256, not here. toprule File sha256

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v3/code/checks_v3.py a4596aa807d57bd7a231a3604237f8ad8d22725e2d55329d2f04acec9a6a2bb0

v3/logs/checks_v3.log b4b7b253b84ace21a725010323050f8067d0d6c855d11e9c02df3edcf3bf47b1

v3/code/build_v3.py 0c5c088fcb7e23e80f2cd0cf1537421c1ea68ec9106b276e2e80db0b1284b91f

v3/code/checks_tex3.py ea6227c5f9d258cdf635024484df7dceab2ade5ca53cef82a8fc44817255a061

v3/inputs/I-B-G-Q-B-v2.tex d9b8fe9ce8c76a5e1b7508adf3ee61475a6c5fc1a1e824bb3f1f6319674b28cb

v4/code/checks_v4.py accb126a9a2f4e6dd0abec4376ebf2bc5a87a2a924064709b1d802f3f65fbddf

v4/logs/checks_v4.log 07a982203e60e23ed4c4af69dcf6a9791dad9674efaece63d7b7c53452499995

v4/code/build_v4.py 65de1da7627deccd46853610fd8553e3c7cffd1344aaa32397ce0165282b4f69

v4/code/checks_tex4.py eec9ddcbb576e6176697d42d6d95d4ef0ea23e9051875badcb67ce45d0cda63d

v4/code/integrate_v4.py a8c7724e65928626be50d7e360b282f624aeb7b13ce4ed36e5216ee73d3d8870

v4/code/meta_v4.py a566d065c1000293d028208ec77f4fefb58b956e5f4411bafb384df4d8ffc3eb

PLAN_t221u.md f1156d7e0f084e48af497cba6b4f1ae0c806f0a38c47c930cdd3cbc5000ff941

v4/inputs/I-B-G-Q-B-v3.tex 08f4f3a033641b259a178156743c5decfab26ff92b4bd90e25254ec9177228e8

v5/code/checks_v5_J.py e6722e1675ca13aefa62631c95bd35fa7b1f66eec1da0c9f8e3dcc4866f3c347

v5/code/checks_v5_O.py c206d1076a2d9bce11d71a0dccb6ca360afd131bc9b838cb8e352cdb94968972

v5/code/checks_v5_Q.py 3ae9b6fa4efeea6e94fb8c83a7f586e8745743723ff68b03348e9b17d4cec308

v5/code/checks_v5_S.py 019aeb75ddc7b29974a0dd86fc2975ba3779af2a29ca75db7bc90c18dc2fc102

v5/code/checks_v5_T.py 2d7b1db7a6318d7c371b6513912892b01990b1da278b19caec609093723d5e6d

v5/code/checks_v5_RV.py 0d33bed72cac5db0ff621b2fe3dfa1cc7b2322452096cfbb3645c275b69f107a

v5/logs/checks_v5_RV.log 38de3d9669ad6da025b1373a107ee13de6c21b726beadbd7a20117c16c061e10

v5/code/build_v5.py a4afb283d77e477ddb4140bc7002d898633621ab62ccd3d9c71ad32f21dc1ee6

v5/code/checks_tex5.py 61baed0c97c39f5fd76ae781c035f6cebd6ebd9784de34d11499c080353a9334

v5/code/integrate_v5.py 3f83f501bccd24cdae580d3451f9ebc65f5571e310a71cece98cf8eb4ec02989

v5/code/meta_v5.py 2703f5263ae107fab5a8167824f7fe80a4126946ae12b41b3125d5b8d5a78da4

PLAN_t223u.md aff65918fae48cd4375d9223f4317901842167efd59fe95363402e72843c21b0

v5/logs/checks_v5_Q.log e491856d7c5fc4c3fba16fbf1a3e01c50ec3e002532cee0a4f1b44fc1309b388

v5/inputs/I-B-G-Q-B-v4.tex ddc3de2477c32088621d6e9211d1d18555dd7f8fcba8cc36e3093655499977b5

v6/code/checks_v6_J.py ada02f9d2dc6c01761c0e659366a45a2e454bd439cdc4f0d952b317022645a6f

v6/code/checks_v6_O.py 2f356217a85b07f1ebcd6accc230ad93949afee71ae306ba77ece062c026b945

v6/code/checks_v6_Q.py ffa26991725958279bfdeb290b30fef1c92bb2a52735f147ee57c2ccbdd52648

v6/code/checks_v6_S.py 31ccb48fe09f536e89b9c79b8ce51e4b97494a2d15972269dec462961a7320e1

v6/code/checks_v6_T.py ba23315201161d4d91d57174a2eb3b9e72b680410a6ba57292e9c9e00369f963

v6/code/c6_o_wick.py ef22ed550ce7140f4e7b3e195d3e442d37ee529cb7b9c3d58c0a2b5cebae1ebd

v6/code/build_v6.py b7b73ecd8ddb0554b770aba6647b9c8fbc09131740206a6b90156089fc73f92e

v6/code/checks_tex6.py e6ead00afc4905a2a354f91beab571668a5567607f597d5fcc659ceceaf35af7

v6/code/integrate_v6.py 29ab8eccb4bf725aac53c4e8dece6481ceb4585d5098067fac7b5f543c93a0c6

v6/code/meta_v6.py 30f56063686c7910af52ac8984e4803b128f8ae391636162f70156774446cd79

v6/logs/vidst_recheck.txt 12e9491528c3b766c9dd7708a34a6e34adf9679e34f019791933f700b578f474

v6/logs/checks_v6_Q.log ffc931cb6b0c54a673d950774fd172973b2d74ab9082d048ebcdf72734e58dec

v6/inputs/I-B-G-Q-B-v5.tex 0d18cfe04d5cf2d29439234203ce83f070f66f91d370c4afd4ce9b0e25efafb4

v3/inputs/draftA_c9cbf1eb.tex c9cbf1ebc8d422e538d632092d27deed150ebb6479d2cab9a46c7db151cd14e9

bottomrule Frozen trees checked before and after the run: XVI-P ec965c15…, XVI-Q 0df9a0d1…, XVI-R 57187bc6… (logs v6/logs/frozen_before.txt, v6/logs/frozen_after.txt); unchanged.

References

[1] N. Cheung, *Black-Hole Information as an Access Problem: Enter, Quantum Recovery, and Complete Residuals*, figshare (2026). DOI: 10.6084/m9.figshare.34335024.v1. https://doi.org/10.6084/m9.figshare.34335024.v1.

[2] N. Cheung, *Operational Wormhole Certificates—Existence, Dimension-Dependent Scaling, Operational Functions, and Composition*, figshare (2026). DOI: 10.6084/m9.figshare.34549575.v1. https://doi.org/10.6084/m9.figshare.34549575.v1.

[3] N. Cheung, *Enter, Quantum Magic, and Quantum Gravity: Readout, Recovery, and the Limits of Geometric Response*, figshare (2026). DOI: 10.6084/m9.figshare.34335582.v1. https://doi.org/10.6084/m9.figshare.34335582.v1.

[4] N. Cheung, *Operational Consistency Does Not Select Lorentzian Geometry*, figshare (2026). DOI: 10.6084/m9.figshare.34441758.v1. https://doi.org/10.6084/m9.figshare.34441758.v1.

[5] N. Cheung, *Realization of Relational Time by Destructive Enter on a Clock Register*, figshare (2026). DOI: 10.6084/m9.figshare.34442907.v1. https://doi.org/10.6084/m9.figshare.34442907.v1.

[6] N. Cheung, ``The Enter Calculus: A Linear Quantum Lambda Calculus with an Operational Characterization

of Measurement," *Frontiers in Physics* **14** (2026), article 1954943. DOI: 10.3389/fphy.2026.1954943. https://doi.org/10.3389/fphy.2026.1954943. Interface/operational background only; not a PDE premise.

[7] N. Cheung, *The Philosophy of Enter: Destructive Readout, Quantum Recovery, and the Refusal of Identification*, Cheung Ngo Medical Limited (2026). Framing essay; not a theorem premise. Programme note: https://enter.grok.me/enter.