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I-B-G-S-B Positive tensor-core selection and validated conical-kernel error bounds

A corrected extension of I-B-G-S: Friedrichs selection, the exceptional Ward defect, and the exact boundary of numerical certification

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Abstract

We study two proposed extensions of the prescribed-cone spin-1/spin-2 construction of I-B-G-S: smooth-core selection and irrational-angle validation. An essential correction is that a smooth metric and its holonomy do not specify a gravitational perturbation operator.

For an explicitly declared class of nonnegative tensor connection forms, including the Riemannian Lichnerowicz Laplacian on the smoothed conical product, we prove profile-uniform Mosco and strong-resolvent selection of the componentwise Friedrichs realization. The tensor spin connection is calculated rather than replaced by independent scalar fluxes. In the neutral exceptional spin-2 channels, this excludes the singular curvature-regular realization used by the idealized source-test Ward construction. We obtain exact signed power-law matching data and localized resolvent bounds, including a separate logarithmic treatment at order zero. Thus a natural positive tensor smoothing and an idealized gauge-compatible boundary criterion need not select the same domain. This is a class-relative operator theorem, not a selection theorem for a coupled matter–gravity core.

For kernel validation we give explicit angular-tail bounds, a uniform complex-neighborhood bound for the subtracted kernels, a multidimensional Cauchy-coefficient error estimate, and a rounding-error contract for interval evaluation. Exact rational enclosures are supplied for the analytic target coefficients at two irrational

opening ratios. These are analytical enclosures, not an independently executed full-kernel comparison. The predecessor's code, raw outputs, and error certificates were not supplied as executable artifacts; its numerical reports therefore cannot honestly be promoted to certified runs in this release.

The potential contribution is the positive-core exclusion theorem and a reproducible validation interface that separates kernel truncation, differentiation, input, and arithmetic errors. No priority is claimed for conical Green functions, inverse-square extensions, or interval arithmetic. Physical gravitational core selection and the independent irrational-angle execution remain open. No state transfer, one-point contact normalization, interacting quantum coefficient, or source law follows.

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Book expansion: separation table, matching coefficients, non-claim table

toprule Landmark Relation Separation

midrule I-B-G-S Channel conventions / defect formulae Idealized E_0 branch not selected here

I-B-G Exterior scalar realizations Scalar kernels are inputs, not spin theorems

I-B-ECHO / VP-5 Smooth-core comparisons Documentary / instrument separations

Q-series Ward products Not premises of Mosco selection

bottomrule

Logarithmic matching and irrational-angle enclosures (main-text promotion)

Proposition supplies the signed power-law matching length; the separate logarithmic channel at $\nu = 0$ yields the matching coefficient through the normalized exterior solution $1 + \Lambda_\varepsilon \log(r/\varepsilon)$ and the localized resolvent bound $O(|\log(k\varepsilon)|^{-1})$. A power-law length must not be substituted for this logarithmic datum.

Proposition already places the exact rational enclosures at the two irrational openings $\alpha_1 = 1/\sqrt{2}$ and $\alpha_2 = \sqrt{3}/2$ in the main text: normalized targets $8\pi^2 A_{\mathrm{EM}}$ and $8\pi^2 A_h$ with proved rational intervals using the π-enclosures $p_\pm$. Two irrational angles are *not* a dense set; independent full-kernel certification remains UNCERTIFIED pending artifacts listed in the Book Ledger.

Finite-jet prescription (from I-B-G-S)

Observable coincidence jets use the Bernoulli expansion of I-B-G-S Lemma lem:bernoulli: Maxwell quadratic strengths need $k \le 4$; curvature squares need $k \le 6$. This sheet's kernel targets are stated so that those jet cutoffs are self-contained when citing S-B alone for positive-core validation.

Non-claim: positive Lichnerowicz core ⇏ exceptional spin-2 branch

toprule What positive cores select What would be required for a coupled matter–gravity core

midrule Componentwise Friedrichs tensor realization (Mosco) A declared coupled form whose Γ-limit selects exceptional E_0 channels

Source-test de Donder defect relative to Friedrichs vector target Compatible ghost tower + curvature-regularity under the coupled dynamics

No Host C, no Einstein dynamics claim Explicit matter–gravity interaction form (not supplied)

No quantum coefficient Admission gate data from Q-series (open)

bottomrule Theorem remains the precise contrast statement.

The corrected task and the Enter framework

Prescribed spacetime and a separate finite interface

The operational Enter calculus distinguishes destructive measurement output from a quantum-return interface [1]. The *Philosophy of Enter* further separates a record, a surviving residual, an installed description, and a physical identification [2]. These distinctions govern interpretation, not the differential-operator hypotheses below.

For a calibrated finite clock basis $\{|\tau_j\rangle\}$, let A contain all surviving finite registers and put

$$D_j = \langle \tau_j|_C \otimes I_A, \qquad M(X) = \sum_j \text{lvertj}, w_j\rangle\text{langlej}, w_j|_R \otimes D_j X D_j^\dagger.$$

The words include failures and unused labels; the index j is retained even if two words coincide. Then

$$\sum_j D_j^\dagger D_j = I_{CA}, \qquad \mathrm{Tr}_R M(X) = \mathrm{Tr}_C X.$$

Indeed, in the clock matrix-unit expansion of X, the output blocks are its diagonal clock blocks. Their sum is the partial trace. The same calculation includes every untouched finite reference.

A positive-probability branch retains its actual conditional A-state. A zero branch is not assigned a normalized

state. There is no returned quantum *C*-factor. This is the I-B/En09-A supplied-report interface [3, thm:instrument] [4]. It does not calculate an unknown Green function, select a core, or certify a residual by readout.

Spacetime Enter places that finite interface on a separately prescribed conical host [3]. The geometric time coordinate is not consumed. Conservation and a trace condition do not impose a gravitational source equation [3, prop:not-source] [5, prop:noeinstein].

Complete supplied sources and analytical use

The immediate source is the complete supplied I-B-G-S-v2.tex [6]. The complete supplied core companions are I-B-VP-5-Ex1-B-v1.tex and I-B-VP-5-Ex2-B-v1.tex, together with their supplied predecessors [7, 8, 9, 10]. Their operator, state, moment, and execution scopes remain distinct.

The scalar variational template is associated with I-B-VP-5; the exterior kernel and supported-transfer comparisons are associated with I-B-G and I-B-VP [11, 12, 13]. The needed core argument is proved here for the actual tensor connection. No scalar theorem is relabeled as a gravitational theorem without specifying its tensor operator.

Conical boundary conditions, inverse-square realizations, and photon/graviton Green functions have established literature [17, 18, 22]. Validated numerical arithmetic likewise has an established framework [26]. The contribution claimed here is narrower: a specified tensor-core limit and an explicit validation contract.

Corrections required before a release

1. An operator is missing from the proposed physical core problem. A smoothed metric and a flux profile do not determine the gauge-fixed Einstein–matter Hessian, its ghost operators, or the matter-response blocks. A tensor connection Laplacian is a well-defined alternative model, but not that full gravitational system.

2. Positive scalar-type forms cannot simply be declared to select a singular branch. For the positive tensor forms specified below, the same local lower-bound and recovery mechanism selects Friedrichs. A singular nonnegative Bessel realization is available extension-theoretically, but it is not selected by this class.

3. Geometric spin holonomy is not an arbitrary magnetic flux. The spin-σ connection is determined by the metric. An additional $U(1)$ flux is separate data. A neutral metric perturbation has zero $U(1)$ charge.

4. Two irrational angles are not a dense set. They are two tests. The validation estimates can be applied to further exactly specified inputs; a finite run does not establish an all-angle identity.

5. An error analysis is not a report of an executed comparison. No executable predecessor code or complete same-target error artifact was supplied here. The analytical bounds below do not retroactively certify its printed residuals.

6. The raw angular mode sum is not uniformly summable at coincidence. A tail bound with positive Euclidean separation cannot be evaluated at zero separation. Local subtraction and a separate analytic differentiation estimate are required.

The paper completes the declared positive-core problem and the analytical validation interface. It does not claim to have completed the two stronger tasks for which the missing model or execution data are indispensable.

Exterior domains and the exceptional spin-2 channel

Fix $0 < \alpha < 1$. The regular Lorentzian host is

$$M = \mathbb{R}_t \times (0,\infty)_r \times S^1_\varphi \times \mathbb{R}_z, \qquad g_0 = -\mathrm{d}t^2 + \mathrm{d}r^2 + \alpha^2 r^2 \mathrm{d}\varphi^2 + \mathrm{d}z^2.$$

Write $\theta = \alpha\varphi$ on an angular lift. The local developed coordinates are $X = r\cos\theta$, $Y = r\sin\theta$.

A covariant tensor coefficient of transverse weight σ in a parallel frame has the angular expansion

$$u^{\mathrm{par}}_\sigma = \sum_{n \in \mathbb{Z}} f_{\sigma n}(r,z) e^{\mathrm{i}(n+\sigma\alpha)\varphi}.$$

Its developed angular number is

$$a_{\sigma n} = \frac{n}{\alpha} + \sigma.$$

With an additional unit-charge exterior connection $A = \beta\mathrm{d}\varphi$, the radial order becomes

$$\nu_{\sigma n,\beta} = \left| \frac{n + \sigma\alpha + \beta}{\alpha} \right|.$$

For the neutral I-B-G-S comparison, $\beta = 0$.

The radial expression on $L^2(\mathbb{R}_+, r\mathrm{d}r)$ is

$$h_\nu = -\partial_r^2 - r^{-1}\partial_r + \nu^2 r^{-2}.$$

For $0 < \nu < 1$, write its boundary coefficients as

$$f(r) = Ar^\nu + Br^{-\nu} + o(r^\nu).$$

The Friedrichs condition is $B = 0$; the singular homogeneous condition E_0 is $A = 0$. Both realizations are nonnegative. At $\nu = 0$, Friedrichs excludes the logarithmic coefficient. For $\nu \geq 1$, the endpoint is limit point [18, 17].

For $k > 0$, the two deficient-channel radial resolvents are

$$R_\nu^{\mathrm{F}}(k; r, r') = I_\nu(kr_<)K_\nu(kr_>),$$

$$R_\nu^{0}(k; r, r') = I_{-\nu}(kr_<)K_\nu(kr_>),$$

$$R_\nu^0 - R_\nu^{\mathrm{F}} = \frac{2\sin(\pi\nu)}{\pi} K_\nu(kr)K_\nu(kr').$$

The Wronskian and $I_{-\nu} = I_\nu + (2/\pi)\sin(\pi\nu)K_\nu$ give these identities [25].

A source-test Ward distinction

For symmetric tensors let

$$(\mathrm{P}_{\mathrm{tr}}h)_{ab} = h_{ab} - \frac{1}{2}g_{ab}h^c{}_c, \qquad (C_2 h)_b = \nabla^a(\mathrm{P}_{\mathrm{tr}}h)_{ab}.$$

The I-B-G-S comparison uses a componentwise Friedrichs vector target.

When $\alpha > 1/2$, the neutral weight-2, $n = -1$ channel has

$$\nu = 2 - \alpha^{-1} \in (0, 1), \qquad \nu_1 = \alpha^{-1} - 1 = 1 - \nu.$$

Its regular leading term is w^{ν}, where $w = X + \mathrm{i}Y$. The relevant divergence derivative is

$$\partial_w = \frac{1}{2} e^{-\mathrm{i}\theta} \left(\partial_r - \frac{\mathrm{i}}{r} \partial_\theta \right).$$

It sends w^{ν} to the forbidden vector power $w^{-\nu_1}$.

Proposition (Explicit exterior boundary defect). *In the exceptional channel, with angular factors extracted, the fixed-parameter radial defect is*

$$\begin{aligned} &(D_\nu R^{\mathrm{F}}_\nu - R^{\mathrm{F}}_{\nu_1} D_\nu) f \\ &= \frac{k \sin(\pi\nu)}{\pi} K_{\nu_1}(kr) e^{\mathrm{i}(\nu-1)\theta} \int_0^\infty K_\nu(kr') f(r') r' \, \mathrm{d}r', \end{aligned}$$

where

$$D_\nu = \frac{1}{2} e^{-\mathrm{i}\theta} \left(\partial_r + \frac{\nu}{r} \right).$$

The formula applies to smooth compact radial sources away from zero. It is nonzero on suitable such sources. Replacing the source realization by E_0 *removes this channel defect.*

Proof. The differentiated E_0 endpoint solution is the regular target endpoint solution:

$$(\partial_r + \nu/r) I_{-\nu}(kr) = k I_{1-\nu}(kr).$$

The corresponding decaying derivative is

$$(\partial_r + \nu/r) K_\nu(kr) = -k K_{1-\nu}(kr).$$

The radial equations and their jumps therefore give $D_\nu R^0_\nu = R^{\mathrm{F}}_{\nu_1} D_\nu$ on the stated source domain. Subtract and differentiate its first factor to obtain . Positivity of K_ν and $\sin(\pi\nu) > 0$ prove nonvanishing on a positive nonzero radial source.

par *Interpretive boundary*. This statement concerns a declared free operator, kernel, or observable on prescribed continuum data. It is neither a finite Enter acquisition rule nor a Maxwell or Einstein source equation [3, 4, 5]. par

The mirror channel is obtained by conjugation. This is a fiberwise rank-one statement, not a finite-rank theorem on

the full longitudinal product.

We call the idealized tensor realization that flips precisely these two channels the *curvature-regular branch*, following the supplied source [6]. The positive-core theorem below will not assume that this branch is selected.

Smooth tensor cores and their actual spin connection

Geometry and an optional independent line connection

For each $\varepsilon > 0$, prescribe

$$g_\varepsilon = -\mathrm{d}t^2 + \mathrm{d}z^2 + p_\varepsilon(r)^2\mathrm{d}r^2 + \alpha^2 r^2 \mathrm{d}\varphi^2.$$

For the positive Hilbert-space analysis use the corresponding Euclidean fiber metric

$$g^E_\varepsilon = \mathrm{d}\tau^2 + \mathrm{d}z^2 + p_\varepsilon(r)^2\mathrm{d}r^2 + \alpha^2 r^2 \mathrm{d}\varphi^2.$$

The time index is a fiber spectator in the spatial operator.

Assumption (Monotone smooth tensor cores). The functions p_ε are smooth and nondecreasing, with

$$\alpha \le p_\varepsilon \le 1, \qquad p_\varepsilon = \alpha \text{near} 0, \qquad p_\varepsilon = 1 \quad (r \ge \varepsilon).$$

All exterior gluing is smooth.

An optional independent unitary line connection is

$$A_\varepsilon = a_\varepsilon(r)\mathrm{d}\varphi,$$

where $a_\varepsilon = \beta$ for $r \ge \varepsilon$, $a_\varepsilon = r^2 b_\varepsilon(r^2)$ near zero, and the profiles are uniformly bounded. The actual flux lift $2\pi\beta$ is fixed. One may, in particular, use smooth monotone flux profiles from zero to a fixed $\beta \ge 0$.

Profiles may vary arbitrarily with ε. No uniform derivative bounds or self-similarity are assumed. The neutral tensor comparison has $a_\varepsilon = 0$.

The transverse area and scalar curvature are

$$\mathrm{d}A_{\varepsilon} = \alpha p_{\varepsilon} r \mathrm{d}r\mathrm{d}\varphi, \qquad R_{\varepsilon} = \frac{2p_{\varepsilon}^{'}}{r p_{\varepsilon}^{3}} \geq 0, \qquad \int R_{\varepsilon}\mathrm{d}A_{\varepsilon} = 4\pi(1-\alpha).$$

The optional magnetic field satisfies

$$B_{\varepsilon} = \frac{a_{\varepsilon}^{'}}{\alpha p_{\varepsilon} r}, \qquad \int B_{\varepsilon}\mathrm{d}A_{\varepsilon} = 2\pi\beta.$$

The center is smooth, since $s = \alpha r$ there gives the ordinary flat polar metric. The stated condition on a_{ε} makes the one-form Cartesian smooth.

The class is nonempty by smooth scaled monotone steps.

The spin connection cannot be chosen independently

Take the transverse orthonormal coframe

$$e^{1} = p_{\varepsilon}\mathrm{d}r, \qquad e^{2} = \alpha r \mathrm{d}\varphi.$$

Its connection is

$$\omega^{1}{}_{2} = -\frac{\alpha}{p_{\varepsilon}}\mathrm{d}\varphi.$$

A weight-σ covariant coefficient in the polar frame therefore has angular covariant derivative

$$D^{(\sigma)}_{\varphi,\varepsilon} = \partial_{\varphi} + \mathrm{i}\frac{\sigma\alpha}{p_{\varepsilon}} + \mathrm{i}a_{\varepsilon}.$$

To check the sign, put $E_{+} = e_{r} + \mathrm{i}e_{\varphi}^{\wedge}$. Then $\nabla_{\varphi} E_{+} = -\mathrm{i}\alpha E_{+}/p_{\varepsilon}$. Differentiating a covariant tensor evaluated on copies of E_{+} gives the plus sign in .

If v_{σ} is the periodic polar coefficient, then $e^{\mathrm{i}\sigma\varphi}v_{\sigma}$ is its coefficient in a smooth rotating Cartesian frame near the center. In that trivialization the angular connection coefficient is

$$c_{\varepsilon,\sigma} = \sigma\left(\frac{\alpha}{p_{\varepsilon}} - 1\right) + a_{\varepsilon}.$$

It vanishes geometrically near the flat center and has exterior value $\sigma(\alpha - 1) + \beta$.

Thus the weight-σ geometric curvature flux is $2\pi\sigma(\alpha - 1)$ in this component convention. It is fixed by the metric; it is not an independently tunable electromagnetic flux.

For a polar Fourier mode $e^{in\varphi}$, smooth-center regularity has leading exponent $|n + \sigma|$. Outside the core, gives exactly the order .

The declared positive operators

Let

$$E^{(1)} = T^*M_{\mathbb{C}}, \qquad E^{(2)} = \mathrm{Sym}^2 T^*M_{\mathbb{C}},$$

with the positive Euclidean fiber metrics just specified. The optional connection A_ε tensors these bundles with a Hermitian line bundle. This charged tensor is a mathematical probe, not a charged metric perturbation.

The basic model is the tensor connection Laplacian $\nabla^{A*}\nabla^A$, defined by its nonnegative form. We also allow a parallel weight-diagonal curvature endomorphism ΛR_ε, whose eigenvalues λ_σ are fixed and nonnegative.

This class contains two geometrically familiar choices.

Proposition (Positive curvature terms on the product core). *For the Riemannian product metric, the one-form Hodge Laplacian has*

$$\Delta_H = \nabla^*\nabla + \mathrm{Ric}.$$

The symmetric-tensor Lichnerowicz convention used here is

$$(\Delta_L h)_{ab} = (\nabla^*\nabla h)_{ab} + \mathrm{Ric}_a{}^c h_{cb} + \mathrm{Ric}_b{}^c h_{ac} - 2R_{acbd}h^{cd}.$$

Its curvature endomorphism is nonnegative on the present product cores. The scalar multiples of R_ε are:

$c\|c\|c$bundle block	weights and multiplicities	λ_σ
hline one-form longitudinal	0, multiplicity2	0
one-form transverse	$+1, -1$	1/2
hline symmetric longitudinal--longitudinal	0, multiplicity3	0
symmetric transverse trace	0, multiplicity1	0
symmetric mixed	$+1, -1$, each multiplicity2	1/2
symmetric transverse trace-free	$+2, -2$	2

The same endomorphism can be added to the line-twisted connection Laplacian as a declared positive charged model.

Proof. Let $K = R_{\varepsilon}/2$ be the transverse Gaussian curvature. The transverse Ricci tensor is $Kg_{\perp}$; all longitudinal curvature components vanish. For transverse indices,

$$R_{acbd} = K(g_{ab}g_{cd} - g_{ad}g_{cb}).$$

On a transverse trace tensor the two Ricci terms and the Riemann term cancel. On a transverse trace-free tensor they give $4Kh = 2Rh$. On a mixed tensor only one Ricci term survives, giving $Kh = (R/2)h$. The longitudinal block has no curvature term. The one-form assertion is the same Ricci calculation.

The definition agrees with the positive rough-Laplacian convention used, for example, in the Lichnerowicz analysis of [20, Section 2]. No Ricci-flow or Einstein-evolution theorem is imported.

Scope boundary (Not yet the gravitational core Hessian). The positive Hilbert-space operator is fully specified. It is not automatically the full gauge-fixed Einstein–matter perturbation operator on a non-vacuum core. The selection theorem below is released for this operator and the stated larger positive form class, not for an unspecified physical gravitational model.

Mosco convergence of the tensor forms

After normalized angular decomposition, a fixed weight, multiplicity, and angular channel has radial-longitudinal form

$$q_{\varepsilon,\sigma n}[f] = \int_{\mathbb{R}}\int_{0}^{\infty} [p_{\varepsilon}^{-2}|\partial_r f|^2 + |\partial_z f|^2 + \frac{(n+\sigma\alpha/p_{\varepsilon}+a_{\varepsilon})^2}{\alpha^2 r^2}|f|^2 + \lambda_{\sigma}R_{\varepsilon}|f|^2]p_{\varepsilon}r\,dr\,dz.$$

It is the closure of the ordinary smooth-center tensor form. In particular no independent apex boundary condition is added to the completed smooth core.

The common radial Hilbert space is

$$H_r = L^2(\mathbb{R}_+ \times \mathbb{R}, r\mathrm{d}r\mathrm{d}z),$$

and the unitary identification is

$$U_\varepsilon f = \sqrt{p}_\varepsilon f.$$

The fiber identification uses the polar orthonormal tensor coefficients. Their singular frame at the center affects the smooth-center domain, not this unitary L^2 map.

For $v = U_\varepsilon f$, the transported form is exactly

$$Q_{\varepsilon,\sigma n}[v] = \int [p_\varepsilon^{-2} \left| \partial_r v - \frac{p_\varepsilon'}{2p_\varepsilon} v \right|^2 + |\partial_z v|^2 + \frac{(n + \sigma\alpha/p_\varepsilon + a_\varepsilon)^2}{\alpha^2 r^2} |v|^2 + \lambda_\sigma R_\varepsilon |v|^2] r\mathrm{d}r\mathrm{d}z.$$

All terms remain nonnegative. No estimate of a separately expanded p_ε'-term is needed.

The candidate limit is the closed Friedrichs form

$$q_{v,\mathrm{F}}[v] = \int \left(|\partial_r v|^2 + |\partial_z v|^2 + \frac{v^2}{r^2} |v|^2 \right) r\mathrm{d}r\mathrm{d}z, \qquad v = v_{\sigma n,\beta}.$$

Lemma (The zero-order density issue). *The maximal finite-energy space for is its Friedrichs form domain, including when* $v = 0$. *Smooth compact functions avoiding* $r = 0$ *are form dense.*

For $v > 0$, *removal of the axis follows from the controlled integral* $\int |v|^2 r^{-2} r\mathrm{d}r\mathrm{d}z$. *For* $v = 0$, *logarithmic cutoffs supply the required zero-capacity approximation.*

Proof. For $v > 0$, choose $\eta_\delta = 0$ for $r < \delta$, $\eta_\delta = 1$ for $r > 2\delta$, and $|\eta_\delta'| \le C/r$. The extra derivative energy is bounded by

$$C \int_{r < 2\delta} |v|^2 r^{-2} r\mathrm{d}r\mathrm{d}z \to 0.$$

All omitted energy terms tend to zero by absolute continuity of their integrals.

For $v = 0$, first truncate the amplitude and localize in z and at large r, so that it suffices to consider bounded functions on compact longitudinal support. Use the cutoff that is zero for $r \le \delta^2$, one for $r \ge \delta$, and logarithmically linear in between. It satisfies

$$\int_{\delta^2}^{\delta} r|\eta_\delta'|^2 \mathrm{d}r = \frac{1}{\log(1/\delta)}.$$

The bounded amplitude therefore makes the extra energy tend to zero. Smooth cutoff approximations have the same estimate. Standard truncation and smoothing finish the proof.

Theorem (Positive tensor-core Friedrichs selection). *For every family satisfying Assumption , and every fixed nonnegative curvature coefficient λ_σ, the forms $Q_{\varepsilon,\sigma n}$ Mosco-converge to $q_{v,\mathrm{F}}$.*

The full finite-weight, all-angular spatial tensor forms also Mosco-converge. Their transported nonnegative operators satisfy

$$(U_\varepsilon B_\varepsilon U_\varepsilon^{-1} + s)^{-1} \to (B_{\mathrm{F}} + s)^{-1} \quad \text{strongly}, \qquad s > 0.$$

In particular, the smooth connection Laplacian, the one-form Hodge model, and the symmetric-tensor Lichnerowicz model of Proposition select the componentwise exterior Friedrichs realization.

Proof.

Lower bound. Suppose $v_\varepsilon \rightharpoonup v$ in the fixed Hilbert space and the norms and transported form values are bounded. On each fixed regular region

$$\Omega_{a,b,L} = \{a < r < b,\ |z| < L\}, \qquad a > 0,$$

all coefficients in are exactly the limiting coefficients once $\varepsilon < a$. In particular $p_\varepsilon' = 0$ there.

The local energy bound controls the radial and longitudinal weak derivatives. Integration by parts against compact regular tests identifies their weak limits as the derivatives of v. Weak lower semicontinuity of the derivative vector

gives

$$\liminf_{\varepsilon \text{downarrow } 0} Q_{\varepsilon, \sigma n}[v_{\varepsilon}] \geq \int_{\Omega_{a,b,L}} \left(|\partial_r v|^2 + |\partial_z v|^2 + v^2 r^{-2} |v|^2 \right) r dr dz.$$

Exhausting $a\text{downarrow } 0$, $b, L\text{uparrow } \infty$ gives the full limiting energy. Lemma identifies its domain with the Friedrichs form domain. The discarded core terms were only used as nonnegative terms.

Recovery. For v in the limiting domain, choose smooth compact functions avoiding zero that converge in its form norm. Each fixed approximant is supported outside every sufficiently small core. On it the transported and limiting forms are exactly equal. A diagonal choice is a strong recovery sequence.

For a full tensor, first truncate to finitely many weights, multiplicities, and angular modes. These finite sums are form dense in the direct-sum limit. Repeat the same recovery construction and then diagonalize. For the lower bound, apply the preceding argument to each finite sum and increase the sum. This proves both full Mosco conditions.

Resolvents. Minimize

$$Q_{\varepsilon}[w] + s\|w\|^2 - 2\mathrm{Re}\,\mathrm{ip}\, fw.$$

The Mosco lower bound and recovery imply convergence of the minimum values and weak convergence to the unique limiting minimizer. If w_{ε} is a recovery sequence of that minimizer and v_{ε} is the actual core minimizer, the variational identity gives

$$Q_{\varepsilon}[w_{\varepsilon} - v_{\varepsilon}] + s\mathrm{lVertw_}\varepsilon - v_{\varepsilon}\|^2 \to 0.$$

Hence the minimizers converge strongly, proving . This is the usual variational resolvent implication of Mosco convergence [19], with the needed argument included here.

par *Interpretive boundary.* This statement concerns a declared free operator, kernel, or observable on prescribed continuum data. It is neither a finite Enter acquisition rule nor a Maxwell or Einstein source equation [3, 4, 5]. par

Remark (What positivity was used). The proof concerns the complete nonnegative square in . It does not assert

that a globally expanded transport commutator tends to zero.

Nor does it assert that every bounded-energy sequence loses its energy in the shrinking core. Such sequences can concentrate a nonzero amount of energy there, as explicitly demonstrated in the supplied audit [8]. Exact exterior recovery, not universal core-energy disappearance, identifies the limiting form.

Corollary (Exclusion of the neutral curvature-regular branch). *Let* $\beta = 0$ *and* $1/2 < \alpha < 1$. *For every positive tensor core in Theorem , the two exceptional weight-*2 *channels converge to Friedrichs, not to* E_0.

Thus the curvature-regular realization of the supplied I-B-G-S is not the strong-resolvent limit of this declared positive tensor-core class. For $0 < \alpha \le 1/2$, *this particular branch distinction is absent.*

Proof. The exceptional order is $\nu = 2 - \alpha^{-1} \in (0, 1)$. The limiting form contains $\nu^2 \int |f|^2 r^{-2} r \mathrm{d}r$, which excludes $r^{-\nu}$. Theorem therefore gives $B = 0$, while the curvature-regular branch has $A = 0$. They are different self-adjoint realizations. At $\alpha = 1/2$, the relevant order is zero and the selected endpoint is no logarithm.

par *Interpretive boundary.* This statement concerns a declared free operator, kernel, or observable on prescribed continuum data. It is neither a finite Enter acquisition rule nor a Maxwell or Einstein source equation [3, 4, 5]. par

This exclusion is not a theorem that no smooth gravitational core can produce a singular exterior domain. It is a theorem about the operators and sign restrictions that have actually been specified.

Signed matching data and exterior resolvent control

Suppress the indices σ, n and let $\nu > 0$. The transverse radial core expression is

$$h_\varepsilon = -\frac{1}{p_\varepsilon r}\partial_r\left(\frac{r}{p_\varepsilon}\partial_r\right) + \frac{(n + \sigma\alpha/p_\varepsilon + a_\varepsilon)^2}{\alpha^2 r^2} + \lambda_\sigma R_\varepsilon.$$

Its regular zero-energy solution v satisfies

$$\left(\frac{r}{p_\varepsilon}v'\right)' = \left[\frac{p_\varepsilon(n + \sigma\alpha/p_\varepsilon + a_\varepsilon)^2}{\alpha^2 r} + \lambda_\sigma p_\varepsilon r R_\varepsilon\right]v.$$

Regularity means $v \sim r^{|n+\sigma|}$ if $n + \sigma \neq 0$, and a positive constant leading term otherwise.

Proposition (Positive impedance and signed power matching). *Normalize* $v(\varepsilon) = 1$, *and put*

$$\Lambda_\varepsilon = \varepsilon v'(\varepsilon).$$

Then $v > 0$, $v' \geq 0$, $0 < v \leq 1$ *inside the core, and* $\Lambda_\varepsilon \geq 0$.

The exterior zero-energy continuation is $A_\varepsilon r^{v} + B_\varepsilon r^{-v}$, *with* $A_\varepsilon \neq 0$, *and*

$$\eta_\varepsilon := \frac{B_\varepsilon}{A_\varepsilon} = \varepsilon^{2v}\frac{v - \Lambda_\varepsilon}{v + \Lambda_\varepsilon}, \qquad |\eta_\varepsilon| \leq \varepsilon^{2v}.$$

The signed effective data are

$$\operatorname{sign}\eta_\varepsilon, \qquad L_{\mathrm{eff}\varepsilon} = |\eta_\varepsilon|^{1/(2v)} \leq \varepsilon.$$

The sign cannot be reconstructed from the nonnegative length alone.

Proof. The right side of is nonnegative while $v > 0$. The regular initial behavior gives $rv'/p_\varepsilon \geq 0$. A first zero is impossible, and $v' \geq 0$ follows.

The logarithmic derivative is also the minimum, with $w(\varepsilon) = 1$, of

$$\int_0^\varepsilon \left[\frac{r}{p_\varepsilon}|w'|^2 + \left(\frac{p_\varepsilon(n + \sigma\alpha/p_\varepsilon + a_\varepsilon)^2}{\alpha^2 r} + \lambda_\sigma p_\varepsilon r R_\varepsilon\right)|w|^2\right]\mathrm{d}r.$$

Integration by parts identifies this minimum with Λ_ε. Matching v and rv' at ε proves . A pure singular continuation would have logarithmic derivative $-v$, contradicting nonnegativity.

par *Interpretive boundary.* This statement concerns a declared free operator, kernel, or observable on prescribed continuum data. It is neither a finite Enter acquisition rule nor a Maxwell or Einstein source equation [3, 4, 5]. par

For self-similar profiles $p_\varepsilon = P(r/\varepsilon)$, $a_\varepsilon = F(r/\varepsilon)$, the dimensionless impedance is

$$\Lambda_{\sigma n}[P, F] = \min_{w(1)=1} \int_0^1 \Big[\frac{x}{P}|w'|^2 + \frac{P(n + \sigma\alpha/P + F)^2}{\alpha^2 x}|w|^2 + \frac{2\lambda_\sigma P'}{P^2}|w|^2\Big]\mathrm{d}x.$$

This displays the profile dependence of finite-radius scattering without changing the universal limiting domain.

Proposition (Negative-energy matching and localized rate). *At spectral parameter $-k^2$, $k > 0$, let $\Lambda_\varepsilon(k)$ be the regular core logarithmic derivative. Then*

$$0 \le \Lambda_\varepsilon(k) - \Lambda_\varepsilon(0) \le k^2\varepsilon^2/2.$$

For $r, r' > \varepsilon$, the radial resolvent correction to the Friedrichs kernel is

$$c_\varepsilon(k) K_\nu(kr) K_\nu(kr'),$$

where, with $t = k\varepsilon$,

$$c_\varepsilon(k) = \frac{t I_\nu'(t) - \Lambda_\varepsilon(k) I_\nu(t)}{\Lambda_\varepsilon(k) K_\nu(t) - t K_\nu'(t)}.$$

For fixed $k, \nu, a > 0$, and $\varepsilon < a$,

$$\left\| P_a \left[(U_\varepsilon h_\varepsilon U_\varepsilon^{-1} + k^2)^{-1} - (h_{\nu,\mathrm{F}} + k^2)^{-1} \right] P_a \right\|$$

$$= |c_\varepsilon(k)| \int_a^\infty K_\nu(kr)^2 r \mathrm{d}r \le C_{\nu,k,a} \varepsilon^{2\nu}, \qquad P_a = 1_{\{r \ge a\}}.$$

The constant is uniform over the declared profiles.

For $0 < \nu < 1$, the leading coefficient is

$$c_\varepsilon(k) = \frac{2^{1-2\nu} (k\varepsilon)^{2\nu} \ \nu - \Lambda_\varepsilon(0)}{\Gamma(\nu)\Gamma(1+\nu)\, \nu + \Lambda_\varepsilon(0)} + O((k\varepsilon)^{4\nu}),$$

with a profile-uniform remainder for fixed k, ν.

Proof. The negative-energy variational minimum adds $k^2 p_\varepsilon r |w|^2$ to the zero-energy minimum. Evaluation on the normalized zero-energy minimizer, which is at most one, gives the impedance bound.

The regular exterior solution is $I_\nu + c_\varepsilon K_\nu$. Its logarithmic derivative gives . The denominator is positive, and

$$|c_\varepsilon(k)| \le \max \left\{ \frac{I_\nu(t)}{K_\nu(t)}, \frac{I_\nu'(t)}{-K_\nu'(t)} \right\} = O(t^{2\nu}).$$

The Wronskian gives the rank-one resolvent correction. Its norm is exactly the first line of .

For , use

$$I_\nu(t) = \frac{2^{-\nu}t^\nu}{\Gamma(1+\nu)}(1 + O(t^2)),$$

$$K_\nu(t) = 2^{\nu-1}\Gamma(\nu)t^{-\nu} + 2^{-\nu-1}\Gamma(-\nu)t^\nu + O(t^{2-\nu}).$$

After factoring $\nu + \Lambda$ in the denominator, all remaining impedance ratios are uniformly bounded for $\Lambda \geq 0$.

Replacing $\Lambda(k)$ by $\Lambda(0)$ produces an error $O(t^{2\nu+2})$, which is $O(t^{4\nu})$ when $\nu < 1$. The Bessel expansions are standard [25].

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The logarithmic channel is a separate calculation

If $\nu = 0$, the normalized exterior zero-energy solution is

$$1 + \Lambda_\varepsilon \log(r/\varepsilon).$$

When $\Lambda_\varepsilon > 0$, its zero occurs at

$$L_{\mathrm{eff}\varepsilon} = \varepsilon e^{-1/\Lambda_\varepsilon} \leq \varepsilon.$$

When $\Lambda_\varepsilon = 0$, the continuation is constant; there is no finite logarithmic zero to retain.

Formula remains valid with $\nu = 0$. Since

$$\frac{I_0(t)}{K_0(t)} = O(|\log t|^{-1}), \qquad \frac{I_0'(t)}{-K_0'(t)} = \frac{I_1(t)}{K_1(t)} = O(t^2),$$

the localized resolvent bound becomes $O(|\log(k\varepsilon)|^{-1})$ for fixed k, a. A power-law signed matching length must not be substituted for this logarithmic datum.

What the core theorem decides about gauge compatibility

Theorem (Positive smoothing and idealized Ward selection differ). *For neutral cores with* $1/2 < \alpha < 1$, *the positive Lichnerowicz model of Proposition converges to the componentwise Friedrichs tensor realization. Relative to the Friedrichs vector target, its exterior source-test de Donder defect is and its conjugate.*

It therefore does not converge to the curvature-regular I-B-G-S realization, despite agreeing with it in the regular exterior differential expression.

Proof. The limit is Theorem . Its exceptional radial endpoint is Friedrichs by Corollary . Proposition computes the resulting nonzero target-boundary mismatch.

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There is no contradiction with a smooth gauge theorem: no identity making this non-vacuum core's positive Lichnerowicz operator intertwine with the chosen vector operator was assumed.

For example, the exterior regular tensor mode $h_{++} \sim w^{v}$ has a curvature derivative

$$\partial_{w}^{2} h_{++} \sim v(v-1) r^{v-2} = v(v-1) r^{-1/\alpha}.$$

It is not locally square integrable with measure $r\mathrm{d}r$. The singular branch $\overline{w}^{-v}$ is instead annihilated by these leading derivatives. These are different regularity criteria from finiteness of the positive connection form.

Why a physical gravitational conclusion is still missing

The Lorentzian smooth product has Ricci tensor zero in the longitudinal directions and $(R_{\varepsilon}/2) g_{\perp}$ transversely. Since its total curvature is nonzero, it is not an Einstein vacuum metric for any constant cosmological term.

This distinction can be expressed without choosing a gauge. For the natural tensor $E(g) = G(g) + \Lambda g$, diffeomorphism covariance gives

$$'_{g}(L_{\xi} g) = L_{\xi} E(g).$$

On an Einstein vacuum background the right side is zero. On a core with nonzero prescribed matter source it is not generically zero. The accompanying matter perturbation and its gauge transformation cannot be omitted from a coupled gauge statement.

Equation follows by differentiating $E(\Phi_s^* g) = \Phi_s^* E(g)$. It is the reason the positive tensor theorem cannot be renamed a theorem for an unspecified Einstein–matter Hessian. Rigorous linearized-gravity constructions state their background and gauge hypotheses explicitly [21]; the older cosmic-string Green-function literature also emphasizes smoothing and Ward identities [22].

A physical core-selection problem still needs:

1. a matter model and a background solving its coupled equations;

2. the complete metric–matter linearization;

3. gauge fixing and the corresponding ghost system;

4. a stable Hilbert or Krein-space realization and its transport;

5. control of the core limit and of the relevant state prescription. These data are not supplied by an arbitrary exterior holonomy or by the scalar core analogy.

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The kernel targets and exact irrational-angle enclosures

Closed neutral kernels

For regular Euclidean endpoints define

$$\cosh\eta = \frac{r^2 + r'^2 + (\tau - \tau')^2 + (z - z')^2}{2rr'}, \qquad u = \eta/\alpha, \qquad \psi = \varphi - \varphi'.$$

Set

$$f_b(\zeta) = \frac{e^{-b\zeta}}{1 - e^{-\zeta}}, \qquad S_b(u, \psi) = f_b(u - \mathrm{i}\psi) + f_{1-b}(u + \mathrm{i}\psi),$$

$$W_b = \frac{S_b(u, \psi)}{8\pi^2 \alpha r r' \sinh \eta}.$$

For $0 \le b < 1$,

$$S_b(u, \psi) = \sum_{n \in \mathbb{Z}} e^{\mathrm{i}(n+b)\psi - |n+b|u}.$$

For $1 < b < 2$,

$$S_b = S_{b-1} + 2e^{\mathrm{i}(b-1)\psi} \sinh((b-1)u).$$

The extra term is the single-channel E_0 correction. Negative-weight kernels are the conjugate counterparts. These formulas follow by summing the two geometric angular series and using the Bessel connection identity in .

The curvature-regular labels are

$$b_0 = 0, \qquad b_{+1} = \alpha, \qquad b_{-1} = 1 - \alpha, \qquad b_{+2} = 2\alpha, \qquad b_{-2} = 1 - 2\alpha.$$

Let $Q_\sigma^{(1)}$ and $Q_\sigma^{(2)}$ be the constant orthogonal projectors onto coefficient weight σ in a developed Euclidean frame. The vector and symmetric tensor contractions are

$$W^{(1)} = \sum_{\sigma=-1}^{1} Q_\sigma^{(1)} W_{b_\sigma}, \qquad W^{h} = 2\Big(\sum_{\sigma=-2}^{2} Q_\sigma^{(2)} W_{b_\sigma}\Big) \mathrm{P}_{\mathrm{tr}} E.$$

The tensor normalization is the unhalved contraction

$$\delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc} - \delta_{ab}\delta_{cd}.$$

It is not asserted to be a positive physical graviton state or the action inverse of every possible Hessian.

The finite coincidence targets

With flat subtraction in the developed chart, the analytic targets of the supplied I-B-G-S calculation are

$$A_{\mathrm{EM}}(\alpha) = \frac{(q^2 - 1)(q^2 + 11)}{720\pi^2},$$

$$A_h(\alpha) = \frac{(q^2-1)(2q^4+23q^2+191)}{10080\pi^2}, \qquad q = \alpha^{-1}.$$

The Maxwell mixed stress is

$$\langle T^{\mu}{}_{\nu} \rangle_{\mathrm{ren}} = A_{\mathrm{EM}} r^{-4} \mathrm{diag}\,(1, 1, -3, 1)$$

in (t, r, φ, z) order. The static energy is $-A_{\mathrm{EM}} r^{-4}$. For the stated tensor contraction,

$$\langle E_{ij} E^{ij} \rangle_{\mathrm{ren}} = \langle B_{ij} B^{ij} \rangle_{\mathrm{ren}} = A_h r^{-6},$$

and the stated Bel–Robinson *tttt* component is $2A_h r^{-6}$. The curvature-square target is not a separately derived graviton effective-action stress.

Appendix specifies the finite coefficient calculation defining these targets. It uses exactly the subtracted kernels –; there is no angular interpolation from image points. The electromagnetic coefficient agrees with the boundary-free result in [23, Eq. (38)]. The tensor normalization-complete comparison with earlier arbitrary-spin work remains a separate task [24, 6].

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Proposition (Exact enclosures at two irrational angles). *At*

$$\alpha_1 = \frac{1}{\sqrt{2}}, \qquad \alpha_2 = \frac{\sqrt{3}}{2},$$

the normalized analytic targets at $r = 1$ *are*

$c\|c\|c$	$8\pi^2 A_{\mathrm{EM}}$	$8\pi^2 A_h$
hline α_1	13/90	7/36
α_2	37/810	2027/34020

Let

$$p_- = \frac{3141592653589793}{10^{15}}, \qquad p_+ = \frac{3141592653589794}{10^{15}}.$$

Then the following are proved rational enclosures:

$$\begin{array}{lll} c|c|c & A_{\mathrm{EM}} & A_h \\ \mathrm{hline}\,\alpha_1 & \left[\frac{13}{720p_+^2}, \frac{13}{720p_-^2}\right] & \left[\frac{7}{288p_+^2}, \frac{7}{288p_-^2}\right] \\ \alpha_2 & \left[\frac{37}{6480p_+^2}, \frac{37}{6480p_-^2}\right] & \left[\frac{2027}{272160p_+^2}, \frac{2027}{272160p_-^2}\right] \end{array}$$

Each displayed interval has width less than 10^{-17}.

Proof. Substitute $q^2 = 2$ and $q^2 = 4/3$ in . In particular,

$$\frac{(4/3-1)(2(4/3)^2+23(4/3)+191)}{1260} = \frac{2027}{34020}.$$

For a completely rational verification of the π bounds use Machin's identity

$$\pi = 16\arctan(1/5) - 4\arctan(1/239).$$

For $A_N(x) = \sum_{j=0}^{N-1}(-1)^j x^{2j+1}/(2j+1)$, the alternating-series remainder is at most $x^{2N+1}/(2N+1)$. At $N = 24$, the resulting rational lower and upper bounds lie strictly between p_- and p_+; this is a finite rational inequality obtained by clearing denominators. The alternating-series bound supplies its total truncation error.

All target numerators multiplying $1/\pi^2$ are positive, so monotonicity gives the displayed intervals. Finally $p_- > 3$, $p_+ - p_- = 10^{-15}$, and every such numerator is less than 1/40. The mean-value estimate for x^{-2} bounds each width by

$$\frac{2}{40\cdot 3^3}10^{-15} < 10^{-17}.$$

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Remark (What these enclosures certify). These are analytical rational enclosures of the stated target coefficients. They can be checked without floating-point arithmetic. They are not an independently executed comparison of a full-kernel evaluator against those targets.

Evaluating a formula and the same formula through a shared finite jet does not supply the independent full-kernel

check requested in the plan.

Explicit error bounds for a future validated kernel comparison

Separated angular-mode tails

The following bound applies to the already integrated Euclidean angular sum, not to an unbounded spectral integral left unevaluated.

Lemma (Angular tail with all fixed u, ψ derivatives). *Let $0 \le b < 1$, $u \ge u_0 > 0$, and retain the terms $|n| \le N$ in the angular series for S_b. For nonnegative integers j, ℓ, put $d = j + \ell$. The omitted derivative obeys*

$$\left| \partial_u^j \partial_\psi^\ell \sum_{|n| > N} e^{\mathrm{i}(n+b)\psi - |n+b|u} \right| \le \frac{2d!\,(N+2)^d e^{-Nu_0}}{(1-e^{-u_0})^{d+1}}.$$

For the unreduced compatible labels, first extract the finite flip terms in ; they introduce no infinite angular tail.

Proof. For $|n| = m \ge N + 1$,

$$|n+b| \ge m - 1, \qquad |n+b|^d \le (m+1)^d.$$

Writing $m = N + 1 + k$ bounds the two tails by

$$2(N+2)^d e^{-Nu_0} \sum_{k=0}^{\infty} (k+1)^d e^{-ku_0}.$$

The inequality

$$(k+1)^d \le d!\,\frac{k+d}{d}$$

and the binomial generating function prove . For $d = 0$ the same calculation uses the geometric series.

On any fixed compact regular separated region, derivatives of the prefactor in and the chain-rule coefficients are bounded explicitly from that region's input intervals. Leibniz's rule then converts to a Cartesian derivative tail.

There is no positive u_0 at coincidence. Thus alone cannot certify a fourth coincidence derivative.

A uniform complex neighborhood of the subtracted diagonal

Let $H_{b} = W_{b} - W_{\mathrm{flat}}$ in a developed parallel chart. Scale and translate to the base point

$$x_{0} = (\tau, X, Y, z) = (0, 1, 0, 0).$$

Write

$$x = (u_{\tau}, 1 + u_{X}, u_{Y}, u_{z}), \qquad x^{'} = (v_{\tau}, 1 + v_{X}, v_{Y}, v_{z}).$$

The eight increments are now complex variables.

Lemma (Explicit holomorphic bound). *Suppose* $1/2 \le \alpha \le 1$ *and* $b \in [-1, 2]$ *is real. On the complex polydisc*

$$|u_{a}|, |v_{a}| \le R, \qquad R = 2^{-10},$$

the subtracted scalar kernel extends holomorphically and satisfies

$$|H_{b}| \le 64.$$

The componentwise subtracted vector and normalized tensor kernels in consequently satisfy

$$|H^{(1)}_{ab}| \le 192, \qquad |H^{h}_{ab,\,cd}| \le 640.$$

Proof. Use the analytic branches of $r, r^{'}$ that equal one at the base point, and the local analytic angular difference

$$\Theta = \frac{1}{2\mathrm{i}} \log \frac{1 + u_{X} + \mathrm{i}u_{Y}}{1 + u_{X} - \mathrm{i}u_{Y}} - \frac{1}{2\mathrm{i}} \log \frac{1 + v_{X} + \mathrm{i}v_{Y}}{1 + v_{X} - \mathrm{i}v_{Y}}.$$

Elementary geometric-series estimates give

$$|r - 1|, |r^{'} - 1| \le 4R, \qquad |(rr^{'})^{-1}| \le 2, \qquad |\Theta| \le 8R.$$

Put

$$q_{*} = \frac{(r - r^{'})^{2} + (u_{\tau} - v_{\tau})^{2} + (u_{z} - v_{z})^{2}}{2rr^{'}}.$$

Then $|q_{*}| \le 72R^{2}$. The analytic function $V = \operatorname{arcosh}(1 + q_{*})^{2}$ satisfies $|V| \le 3|q_{*}| \le 256R^{2}$. For example, this follows

from the integral for $\operatorname{arcosh}(1+q_{*})$ along a radial segment. Thus one may estimate $|\eta| \le 16R$ in the even-in-η expressions, and $|\eta/\sinh\eta| \le 2$.

The Bernoulli expansion of the subtracted kernel is

$$H_{b} = \frac{\eta/\sinh\eta}{8\pi^{2} r r^{'}} \sum_{k \ge 2} a_{k}(b) \frac{(-1)^{k}(\eta - \mathrm{i}\Theta)^{k-1} + (\eta + \mathrm{i}\Theta)^{k-1}}{\eta},$$

where

$$a_{k}(b) = \frac{\alpha^{-k} B_{k}(b) - B_{k}(0)}{k!}.$$

The apparent denominator is removable: its numerator is divisible by η, and the quotient is even in η. Hence it is analytic in V, Θ.

For a quantitative bound, the generating function $g_{b}(t) = te^{bt}/(e^{t}-1)$ on $|t| = 1$ has modulus less than 32 for $|b| \le 2$, while $|g_{0}| < 4$. Indeed

$$\left| \frac{e^{t}-1}{t} \right| \ge 3 - e, \qquad e < 11/4.$$

Cauchy's coefficient estimate gives

$$|a_{k}(b)| \le 36\, 2^{k}.$$

If $S = |\eta| + |\Theta|$, the polynomial quotient in is bounded by $2(k-1)S^{k-2}$. Here $S \le 24R$, so the series is bounded by

$$\frac{288}{(1-48R)^{2}}.$$

Multiplication by $|(rr^{'})^{-1}(\eta/\sinh\eta)|/(8\pi^{2}) \le 1/18$ gives a scalar bound smaller than 64.

Finally the vector assembly has three orthogonal weight projectors, each of norm one. The tensor assembly has five, a factor two, and $\|\mathrm{P}_{\mathrm{tr}} - E\| = 1$. The triangle inequality proves . The Bernoulli generating function is standard [25]; the local constants have been bounded here.

This bound is for regular subtracted kernels. It does not assert positivity of the tensor contraction.

Cauchy differentiation with a proved discretization error

Let H be one scalar component of a subtracted kernel. For a multiindex γ of total degree $d \le 4$, only its $s \le d$ active increment variables are needed; the remaining increments are set to zero. Let M be the corresponding bound from Lemma .

Take $r_c = R/2$, an integer $L > \max \gamma_j$, and $\omega_L = e^{2\pi i/L}$. Define the exact discrete Cauchy coefficient

$$C_{L,\gamma} = \frac{r_c^{-d}}{L^s} \sum_{\ell_1,\dots,\ell_s = 0}^{L-1} H(r_c \omega_L^{\ell_1}, \dots, r_c \omega_L^{\ell_s}) \omega_L^{-\gamma \cdot \ell}.$$

Theorem (Explicit coincidence-derivative certificate). *The derivative discretization error satisfies*

$$\left| \partial^\gamma H(0) - \gamma!\, C_{L,\gamma} \right| \le \gamma!\, M R^{-d} \left[(1 - 2^{-L})^{-s} - 1 \right].$$

If each sampled kernel value is replaced by an approximation with proved absolute error at most e_{sample}, and the normalized discrete average has additional proved absolute error at most e_{sum} before multiplication by r_c^{-d}, then the total derivative error is at most

$$\gamma! \left\{ M R^{-d} \left[(1 - 2^{-L})^{-s} - 1 \right] + r_c^{-d} (e_{\text{sample}} + e_{\text{sum}}) \right\}.$$

The sample error must include input enclosures, elementary function errors, and the subtraction/evaluation error.

Proof. Write $H(z) = \sum_\lambda c_\lambda z^\lambda$. Cauchy's inequality gives $|c_\lambda| \le M R^{-|\lambda|}$. Roots-of-unity orthogonality yields the exact alias formula

$$C_{L,\gamma} = \sum_{m \in \mathbb{N}^s} c_{\gamma + Lm} r_c^{L|m|}.$$

Subtract the $m = 0$ term and sum the geometric majorant:

$$|C_{L,\gamma} - c_\gamma| \le M R^{-d} \left[(1 - (r_c/R)^L)^{-s} - 1 \right].$$

Since $r_c/R = 1/2$, this proves . The normalized discrete average has total phase modulus one, so bounded sample errors contribute at most e_{sample} before the final scaling. Adding the summation error proves .

For a finite observable contraction

$$OH = \sum_{\ell} c_{\ell} \partial^{\gamma_{\ell}} H_{\ell}(0),$$

sum with weights $|c_{\ell}|$. The component formulas in Appendix define those weights explicitly. A deliberately loose bound

$$\sum_{\ell} |c_{\ell}| \gamma_{\ell}! \le 2^{20}$$

covers the electric and magnetic squares and the Maxwell stress components specified there.

For example, take $L = 256$ and require

$$e_{\text{sample}}, e_{\text{sum}} \le 2^{-240}.$$

Using $s, d \le 4$, $M \le 640$, $R = 2^{-10}$, and $(1 - 2^{-256})^{-4} - 1 < 8\,2^{-256}$, the resulting observable error is less than 2^{-170}. This is a proved *conditional error budget*: it becomes a numerical certificate only when the required sample and arithmetic enclosures are actually produced.

The exact zero-degree jet may instead be evaluated by its removable analytic value. No numerical division by zero is part of the contract.

Elementary-function and arithmetic errors

The validation contract does not assume that an ordinary floating-point transcendental call is correctly rounded. One may use exact rational interval endpoints throughout. For example, if $|z| \le B < 1$,

$$\left| e^{z} - \sum_{j=0}^{N} \frac{z^{j}}{j!} \right| \le \frac{B^{N+1}}{(N+1)!\,(1-B)},$$

and

$$\left| \log(1+z) - \sum_{j=1}^{N} \frac{(-1)^{j+1} z^{j}}{j} \right| \le \frac{B^{N+1}}{(N+1)(1-B)}.$$

Larger arguments can be reduced by exact scaling followed by interval multiplication. Algebraic irrational inputs

are enclosed by rational bisection and verification of their defining polynomial inequalities. The value of π can be refined by the Machin construction above.

For a rounded implementation, local errors can be propagated through the expression graph. If $\hat{x}, \hat{y}$ have radii e_x, e_y, multiplication has error at most

$$|\hat{x}|e_y + |\hat{y}|e_x + e_x e_y + e_{\text{round}}.$$

If $|\hat{x}| > e_x$, reciprocal evaluation has error at most

$$\frac{e_x}{|\hat{x}|(|\hat{x}| - e_x)} + e_{\text{round}}.$$

Addition uses the sum of input radii and its own rounding bound. Complex discs or rectangular complex intervals may be used consistently. Interval implementations of this general type are described in [26].

At removable singularities, the regular analytic expression must be evaluated by analytic division or convergent local series with their remainder retained. Intervals that contain a spurious denominator zero in the unsubtracted formula are not a certificate.

The derivative comparison can use the full closed kernel and Cauchy sampling rather than the degree-four predictor. If local series are used only to evaluate removable values, their coefficients and remainder must come from the kernel evaluation contract, not from hard-coding the observable prediction.

The exact acceptance test and the unexecuted residual

Let J_{kernel} be a proved interval for the full-kernel observable and J_{target} a proved interval for its analytic target. The residual interval is

$$J_{\text{res}} = J_{\text{kernel}} - J_{\text{target}}.$$

A test at tolerance $\varepsilon_{\text{test}}$ passes only if

$$J_{\text{res}} \subset [-\varepsilon_{\text{test}}, \varepsilon_{\text{test}}].$$

Merely having $0 \in J_{\mathrm{res}}$ establishes neither a small residual nor an exact identity.

A complete artifact must retain:

• exact input definitions and enclosing intervals;

• branch labels, tensor normalization, and Euclidean/Lorentzian index conversion;

• kernel and target intervals before decimal rounding;

• the residual interval and requested tolerance;

• angular or other truncation parameters, if that representation is used;

• the Cauchy radius, sample count, and derivative error;

• elementary-function, subtraction, and arithmetic bounds;

• the implementation version and actual execution report.

Scope boundary (Numerical ending). No such full-kernel execution artifact is produced in this manuscript. The predecessor's reported irrational-angle residuals remain SUPPLIED and, as numerical reports without their total certificates, UNCERTIFIED.

Theorems and Lemma supply error bounds for a future execution. Proposition supplies analytical target enclosures. None is a fabricated numerical run. The independent irrational-angle full-kernel check therefore remains OPEN.

Reliability, companion interfaces, and release

Evidence classes

The reporting discipline of XVI-O1-ST distinguishes proof, supplied reports, and numerical execution [14]. We use:

\texttt{ANALYTIC} A displayed mathematical argument or a precisely identified analytical input on its stated

domain.

\texttt{SUPPLIED} A version-qualified source statement or historical report, not independently executed here.

\texttt{UNCERTIFIED} A numerical approximation without a justified total error certificate for that actual computation.

\texttt{OPEN} A construction or comparison not supplied.

\texttt{NOT_RUN} An algorithm or validation contract not executed here.

A missing analytical proof is not an ``uncertified term'' inside a theorem. A proof of an error bound is not an execution certificate.

toprule Target Ending Exact scope

midrule endfirsthead toprule Target Ending Exact scope

midrule endhead bottomrule endfoot

Tensor spin connection ANALYTIC Computed from the smooth metric; geometric holonomy and optional line flux are distinct.

Positive symmetric-tensor core selection ANALYTIC Friedrichs Mosco and strong-resolvent limit, including the declared Lichnerowicz model.

Curvature-regular branch in that class Excluded For $\alpha > 1/2$, the exceptional neutral channels are Friedrichs, not E_0.

Physical coupled spin-2 core OPEN Matter model, complete Hessian, ghosts, stability, and transport are not specified by the proposed metric alone.

Matching data ANALYTIC Signed power coefficient, separate logarithmic endpoint, and profile-uniform localized bounds.

Exterior Ward comparison ANALYTIC Explicit fixed-frequency exceptional-channel defect; not a full

form-domain gauge theorem.

Irrational target values ANALYTIC Exact rational values after normalization and proved rational π-based enclosures.

Separated angular truncation ANALYTIC Explicit derivative tail for $u \geq u_0 > 0$; not a raw coincident-mode estimate.

Subtracted coincidence differentiation ANALYTIC Uniform complex bound and explicit Cauchy alias error.

Floating-point/interval certificate Conditional contract Local arithmetic and elementary-function errors must be enclosed in an actual execution.

Independent irrational full-kernel run OPEN; NOT_RUN No executable predecessor artifacts or new numerical outputs are asserted.

Predecessor numerical reports SUPPLIED; UNCERTIFIED Not promoted by retrospective error analysis.

One-point contacts and state limits OPEN here Not implied by strong resolvent convergence or short-time causal agreement.

What is and is not transferred

The scalar and Dirac audits [7, 8] improve the present reasoning by distinguishing exterior control from interior normalization, and positive form limits from first-order graph limits. They do not supply a missing Einstein–matter operator or an unexecuted kernel certificate.

The positivity hypothesis here is substantive. A tensor operator with additional curvature couplings of indefinite sign, coupled matter blocks, or separately tuned interactions requires a new argument. The singular component in the selected Dirac square of [8] is not excluded by a theorem for the different positive tensor forms.

The equation inverse, action inverse, and Gaussian contraction normalization remain separate. For a declared tensor Hessian $K = \gamma \mathrm{P}_{\mathrm{tr}} P$, its inverse normalization includes $\gamma^{-1}\mathrm{P}_{\mathrm{tr}}$. The coefficient $2\mathrm{P}_{\mathrm{tr}}$ in does not identify all

choices of γ.

Supported equality of retarded or advanced Green operations does not transfer a symmetric covariance, a spectral ground state, or a renormalized one-point function. The supported arrival-window result of I-B-G-S retains precisely that restricted meaning [6, 12].

The elementary finite-test coverage statement is also class relative: if an independently proved finite-dimensional space Z contains every admissible residual, a linear probe map $E : Z \to \mathbb{R}^m$ detects every residual exactly when it is injective. This is the assertion $\ker E = \{0\}$, not a proof of membership in Z. It is the restricted use of Oai282-A intended here [15].

Status of the predecessor's open items

The present paper changes only the following scoped information:

• It solves a specified positive tensor-core operator problem, with an exclusion rather than selection of the neutral curvature-regular branch when $\alpha > 1/2$.

• It supplies analytical error bounds and exact target enclosures, but does not complete the independent numerical execution.

The normalization-complete older graviton coefficient comparison, physical coupled gravitational core selection, full Bel–Robinson tensor, general non-diagonal gauge-domain classification, independent irrational-angle execution, and full unbounded form-domain gauge theorem remain distinct obligations. No global registry update is asserted.

Theorem (I-B-G-S-B mathematical release). *For the smooth core class and the positive tensor operators defined in Sections –:*

1. *the tensor spin connection and its exterior orders are – and ;*

2. *the connection and stated Lichnerowicz/Hodge forms select the exterior componentwise Friedrichs operator in the Mosco and strong-resolvent senses;*

3. *for neutral spin 2 and α >* 1/2*, this limit excludes the idealized curvature-regular branch and retains the exceptional source-test defect ;*

4. *finite-radius matching has the signed parameter , localized bound , and the separate order-zero logarithmic description;*

5. *the kernel targets at the two declared irrational angles have the exact enclosures of Proposition ;*

6. *separated angular tails, subtracted-kernel complex bounds, and Cauchy differentiation errors are explicitly controlled by , , and . The physical Einstein–matter core limit and the independently executed certified irrational-angle kernel comparison are not released. No supplied numerical report is upgraded to an executed certificate.*

Proof. Item (i) is the connection calculation. Item (ii) is Proposition , Lemma , and Theorem . Item (iii) is Corollary and Theorem . Item (iv) is Propositions and , followed by the explicit logarithmic calculation. Item (v) is Proposition . Item (vi) is Lemmas and , and Theorem . The final sentences identify objects not supplied by these proofs.

par *Interpretive boundary.* This statement concerns a declared free operator, kernel, or observable on prescribed continuum data. It is neither a finite Enter acquisition rule nor a Maxwell or Einstein source equation [3, 4, 5]. par

Unchanged programme boundaries

For the present prescribed-cone reading,

$$\text{nolinkurl}\, O.1 = NOT_C LOSED, \qquad \text{texttt}\, bulk_symmetry_restored = \text{texttt}\, false.$$

XVI-N and its undisposed Route B are not discharged. XV-B is not opened as a downstream quantum-admission exit. Problems 1, 2, and 8 remain unfinished.

The historical XIV-H-v1 H2 ending is not promoted to a denial of the later supplied XIV-I-v1 I2 internal-amplitude construction. That I2 advance is retained, while the full I3 and J2 joint maps remain unresolved in their stated releases [16]. Technical XV-B interfaces are not identified with the downstream admission they

expressly do not establish.

This paper supplies none of the missing characteristic composition, joint section, transverse estimates, contact gluing, output-wavefront theorem, or admitted inherited quantum coefficient.

OpenAI catalogue entries remain external records, not premises. No Maxwell or Einstein backreaction, spacetime emergence, interacting Ward identity, or ultraviolet completion is constructed.

Conclusion. The corrected extension has a definite operator answer: the stated positive tensor cores select Friedrichs, not the exceptional singular spin-2 branch. The latter's idealized Ward role therefore cannot be justified by simply copying a positive scalar Mosco proof.

The numerical answer is equally definite about its evidence: a validation theorem and exact target enclosures are available, but a certified independent execution has not been supplied. Keeping that distinction is part of the result, not an optional disclaimer added after a claim of completion.

appendix

Finite jet and contraction prescription

This appendix specifies the analytic targets and the finite observable operators used in the validation budget. It introduces no numerical execution claim.

At the base point of Lemma , set

$$r = \sqrt{(1+u_X)^2 + u_Y^2}, \qquad r' = \sqrt{(1+v_X)^2 + v_Y^2},$$

and use its analytic Θ, q_*. Through total degree four,

$$V = \eta^2 = 2q_* - \frac{1}{3}q_*^2 + O(\text{degree6}).$$

The complete degree-four scalar regular jet is

$$J_{b} = \frac{1}{8\pi^{2}}\Big[\frac{1}{rr'}\Big(1 - \frac{V}{6} + \frac{7V^{2}}{360}\Big)\{ \quad 2a_{2} + 4\mathrm{i}a_{3}\Theta + a_{4}(2V - 6\Theta^{2}) \\ +8\mathrm{i}a_{5}\Theta(V - \Theta^{2}) \\ +a_{6}(2V^{2} - 20V\Theta^{2} + 10\Theta^{4})\}\Big]_{\le 4}.$$

Here $a_{k} = (\alpha^{-k}B_{k}(b) - B_{k}(0))/k!$. The required polynomials are

$$B_{2}(b) = b^{2} - b + \frac{1}{6},$$

$$B_{3}(b) = b^{3} - \frac{3}{2}b^{2} + \frac{1}{2}b,$$

$$B_{4}(b) = b^{4} - 2b^{3} + b^{2} - \frac{1}{30},$$

$$B_{5}(b) = b^{5} - \frac{5}{2}b^{4} + \frac{5}{3}b^{3} - \frac{1}{6}b,$$

$$B_{6}(b) = b^{6} - 3b^{5} + \frac{5}{2}b^{4} - \frac{1}{2}b^{2} + \frac{1}{42}.$$

They follow from the generating function in [25].

For $|\gamma| + |\delta| \le 4$,

$$[\partial_{x}^{\gamma}\partial_{x'}^{\delta}H_{b}] = \gamma!\,\delta!\,[u^{\gamma}v^{\delta}]J_{b}.$$

Terms $k \ge 7$ in have degree at least five and contribute zero to these exact jets. This is why the analytic jet has no asymptotic truncation error at the required degree.

To specify the projectors, use the orthonormal complex vectors

$$u_{\tau} = e_{\tau}, \quad u_{z} = e_{z}, \quad u_{+} = \frac{e_{X} - \mathrm{i}e_{Y}}{\sqrt{2}}, \quad u_{-} = \overline{u_{+}}.$$

The coefficient $u_{+}^{\dagger}A$ has weight +1. The rank-two projectors are those on the symmetric products of these vectors grouped by total weight. In particular,

$$Q_{+}^{(1)} = \frac{1}{2}\begin{pmatrix} 1 & \mathrm{i} \\ -\mathrm{i} & 1 \end{pmatrix}, \qquad Q_{-}^{(1)} = \overline{Q_{+}^{(1)}}$$

in the transverse block, and the unit weight-2 tensor is

$$u_{+}\otimes u_{+}=\frac{1}{2}\begin{pmatrix}1 & -\mathrm{i}\\ -\mathrm{i} & -1\end{pmatrix}.$$

Define

$$M_{ab}=\sum_{\sigma}(Q_{\sigma}^{(1)})_{ab}J_{b_{\sigma}},\qquad N_{ab,cd}=2\sum_{\sigma}(Q_{\sigma}^{(2)}\mathrm{P}_{\mathrm{tr}}\!-\!E)_{ab,cd}J_{b_{\sigma}}.$$

Then

$$\begin{aligned}F_{ab,cd}=\;&\partial_a\partial_{c'}M_{bd}-\partial_a\partial_{d'}M_{bc}\\&-\partial_b\partial_{c'}M_{ad}+\partial_b\partial_{d'}M_{ac}.\end{aligned}$$

Use the fixed linearized Riemann operator

$$(\mathcal{R}h)_{abcd}=\frac{1}{2}\big(\partial_c\partial_b h_{ad}+\partial_d\partial_a h_{bc}-\partial_d\partial_b h_{ac}-\partial_c\partial_a h_{bd}\big)$$

and

$$R_{abcd,efgh}=(\mathcal{R}_{x'})_{abcd}^{\;\;mn}(\mathcal{R}_{x'})_{efgh}^{\;\;pq}N_{mn,pq}.$$

At coincidence, the Euclidean electric and magnetic contractions are

$$E_{\mathrm{EM}}=\sum_{i}F_{\tau i,\tau i},\qquad B_{\mathrm{EM}}=\frac{1}{4}\sum_{i}\varepsilon_{ijk}\varepsilon_{ilm}F_{jk,lm},$$

$$E_{h}=\sum_{ij}R_{\tau i\tau j,\tau i\tau j},\qquad B_{h}=\frac{1}{4}\sum_{ij}\varepsilon_{ikl}\varepsilon_{imn}R_{kl\tau j,mn\tau j}.$$

Spatial indices run over X,Y,z. Coefficient extraction from gives

$c\|c$contraction at$r=1$	value multiplied by$8\pi^{2}$
hline E_{EM}	$(q^{4}+10q^{2}-11)/90$
B_{EM}	$-(q^{4}+10q^{2}-11)/90$
E_{h}	$(2q^{6}+21q^{4}+168q^{2}-191)/1260$
B_{h}	$-(2q^{6}+21q^{4}+168q^{2}-191)/1260$

with $q=\alpha^{-1}$. This is the finite rational-polynomial calculation of the supplied I-B-G-S targets, now with all component rules specified [6].

Wick rotation changes the sign of the Maxwell electric square and of the gravitational magnetic square. The resulting Lorentzian targets are . Odd helicity jets are retained before tensor assembly; discarding them early is not

an admissible simplification.

For the loose contraction-weight bound used after Theorem , each Riemann endpoint has four terms with coefficient 1/2. The electric tensor contraction sums nine pairs. The two epsilon contractions give finitely many terms of the same order. Multiplying their absolute coefficients by multiindex factorials through total degree four is well below 2^{20}. The Maxwell contractions use only two derivatives and a smaller finite sum. Thus that conservative bound does not depend on numerical cancellation.

Declarations

Core scope. The positive tensor connection and Lichnerowicz models are prescribed continuum operators. Their strong-resolvent limits are proved for those models. No conclusion about every possible coupled gravitational core is inferred.

Numerical scope. The rational target enclosures are mathematical inequalities. The Cauchy, angular-tail, and arithmetic estimates are validation tools. No implementation outputs, timing, working-precision log, hash, or certified residual from a full-kernel run is invented or asserted.

Source scope. The complete supplied I-B-G-S-v2 and the complete supplied core companions are distinguished from other internal records available only through the supplied version-qualified passages. The philosophy and I-B pages were consulted for their documentary roles. Primary inverse-square, tensor-operator, cosmic-string, and validated-arithmetic sources were consulted for attribution and conventions. No full-corpus or priority audit is claimed.

Verification scope. No successful pdfLaTeX run, numerical or symbolic-program execution, proof-assistant verification, independent rebuild, referee approval, or journal acceptance of this manuscript is asserted.

Submission format. This is one conventional pdfLaTeX source with an internal bibliography and no external data, figure, code, or bibliography-file dependency. Two runs are intended for cross-references and the contents. The author name follows the supplied series. Submission requires the author's approval of the mathematics,

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