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I-B-G-S: Intrinsic spin-1 and spin-2 Green operators on the prescribed conical spacetime

Helicity domains, apex Ward obstructions, curvature regularity, and supported transfer Version 2: domain, normalization, and evidence audit

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Abstract

We construct extension-dependent free spin-1 and spin-2 Green operators on the prescribed conical product of I-B and examine their compatibility with gauge divergence. The central result is a domain distinction invisible to the local differential expression: the componentwise Friedrichs vector operator is compatible with the scalar Friedrichs ghost, whereas the componentwise Friedrichs symmetric-tensor operator fails the de Donder intertwining identity when the deficit is less than π. At fixed longitudinal frequency the latter failure is an explicit radial rank-one boundary defect in each of two helicity channels. Within the stated channel-diagonal, scale-invariant family, replacing precisely those channels by their singular nonnegative Bessel realization restores the source-test Ward identities and finite local curvature for smooth spectral packets. We call this the curvature-regular branch; it is a mathematical selection on the idealized cone, not a physical core-limit theorem.

Closed Sommerfeld–Dowker kernels give finite propagation, localization of the branch difference to the apex-diffracted region, and equality with completion Green operations before additional arrivals. This equality does not identify symmetric covariances or states. A finite Bernoulli-jet prescription yields the Maxwell stress tensor and, for a separately declared graviton-contraction normalization, the electric and magnetic curvature squares. The regular Ricci–Riemann correlator vanishes in the compatible branch. The potential significance is an

explicit demonstration that positivity and componentwise regularity do not alone determine a gauge-compatible apex domain. The full form-domain theorem, spin-2 core selection, complete Bel–Robinson tensor, and independent irrational-angle graviton check remain open. No interacting apex algebra, interacting BRST differential, Einstein dynamics, backreaction, or emergence is constructed.

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Book edition note (t427u)

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Book expansion: separation table, defect, Mosco, Q-separation

toprule Landmark Relation Separation

midrule I-B-G Scalar SCD / positivity / E_0 Exact theorem labels cited; not rewritten

I-B-G-S-B Positive-core Mosco selection Idealized branch not selected by those cores

I-B-ECHO Channel conventions Finite instruments not premises of defect formulae

I-B-G-Q series Source-test Ward language Source-test Ward ≠ interacting product

I-B-VP Hadamard / stress comparisons Documentary; not a Green-operator premise

bottomrule

Rank-one radial defect (both helicities)

The explicit radial formula displays the rank-one defect without an unspecified Wronskian constant. Book edition emphasis: the same radial rank-one structure applies in both exceptional helicity channels of the curvature-regular branch (Definition); the defect is a full-product obstruction when read as a product-level Ward failure, not merely a mode-label curiosity. Lemma (Bernoulli-jet contraction) is the numbered finite-jet tool used for Maxwell ($k \le 4$) and curvature-square ($k \le 6$) coincidence jets.

Why positive cores miss the idealized branch

I-B-G-S-B proves profile-uniform Mosco / strong-resolvent convergence of natural positive tensor cores (including the Riemannian Lichnerowicz Laplacian on the smoothed cone) to the *componentwise Friedrichs* realization. Consequently the idealized curvature-regular spin-2 branch of this sheet—which uses E_0 in exceptional channels—is *not* selected by those cores. This sheet retains the idealized branch as a mathematical object; S-B records the positive-core selection theorem. Separation: neither statement admits a quantum coefficient or creates Host C.

Separation from the Q-series

A Maxwell source-test Ward identity on free Green operators is a *test* of an already constructed inverse. It is not an interacting characteristic product, not a renormalized Ward hierarchy, and not an RG-closed certificate. The Q-series treats those missing objects as open gates; this sheet does not import them as premises.

Purpose, revision scope, and the Enter boundary

The analytical question

I-B prescribes a regular conical spacetime and a separate finite operational interface [3]. I-B-G constructs scalar Green operators only after a spatial realization has been supplied; it explicitly distinguishes a scalar inverse from a compatible Maxwell or gravitational gauge domain [5]. The present paper addresses that distinction.

On the regular cone, a parallel-frame gauge-fixed wave operator acts componentwise as a scalar wave operator. This observation does not settle the boundary condition at the missing apex. A derivative can carry a permitted radial branch into a branch excluded by the ghost realization. The resulting boundary defect is the issue studied here.

The supplied predecessor, I-B-G-S.tex, is cited as a source record [6]. This revision makes its channelwise arguments explicit and separates them from the unproved full form-domain statement. It also separates:

• a holonomy class from an unreduced kernel label;

• a scalar equation inverse from an action inverse;

• a gauge-fixed Gaussian contraction from a positive physical state;

• a radial finite-rank defect from the rank of an operator on the full product Hilbert space;

• written algebraic identities from reported computational checks.

Enter and spacetime Enter

The operational background is Cheung's Enter calculus [1] and the interpretive distinctions of *The Philosophy of*

Enter [2]. The latter is an introductory essay, not a theorem premise.

For a finite clock C, an orthonormal calibration basis $\{|\tau_j\rangle\}$, and a surviving register A, put

$$D_j = \langle \tau_j|_C \otimes I_A, \qquad \mathscr{M}(X) = \sum_j \mathrm{proj}\, j_R \otimes D_j X D_j^\dagger.$$

Then

$$\sum_j D_j^\dagger D_j = I_{CA}, \qquad \mathrm{Tr}_R\, \mathscr{M}(X) = \mathrm{Tr}_C\, X,$$

including untouched-reference extensions. These are the complete destructive-report identities of I-B and En09-A [3, thm:instrument] [4].

This instrument returns a classical code and the complete specified residual interface. It does not supply a continuum Green operator, a gauge-domain identity, a state, a renormalized tensor, or an unknown field value. Consumption concerns finite quantum names, not the geometric time axis. None of the analytical results below is extracted from that finite channel.

Non-sourcing and standing boundaries

The cone, deficit, worldsheet, field normalization, apex realizations, and subtraction convention are inputs. A conserved traceless tensor is not thereby a source of the cone: this is the I-B prop:not-source and X-A-ST prop:noeinstein distinction [3, 9].

Throughout this paper:

$$\text{O.1 on the cone} = \texttt{NOT_CLOSED}, \qquad \texttt{bulk_symmetry_restored} = \texttt{false}.$$

XVI-N is not discharged. The downstream XV-B quantum-admission exit is not opened. The characteristic-product programme remains separate from the free inverses constructed here. No external catalogue entry is an analytical premise.

Host, signature, and normalization ledger

Assumption (Prescribed host). Fix

$$0 < \mu < 2\pi, \qquad \alpha = 1 - \frac{\mu}{2\pi} \in (0,1), \qquad c = 1.$$

Let

$$M = \mathbb{R}_t \times (0,\infty)_r \times S^1_\varphi \times \mathbb{R}_z, \qquad g = -\mathrm{d}t^2 + \mathrm{d}r^2 + \alpha^2 r^2 \mathrm{d}\varphi^2 + \mathrm{d}z^2.$$

The metric completion adds

$$\mathscr{W} = \mathbb{R}_t \times \{o\} \times \mathbb{R}_z.$$

All field kernels and ordinary differential operations in this paper have regular endpoints $r, r' > 0$.

Write $\theta = \alpha\varphi$ on an angular lift. The local developing map is

$$\mathrm{dev}\,(t,r,\varphi,z) = (t,X,Y,z) = (t, r\cos\theta, r\sin\theta, z).$$

The physical developed metric is $\eta_- = \mathrm{diag}\,(-1,1,1,1)$. For comparison with the ST completion convention, set $\eta_+ = -\eta_-$. Then

$$\mathrm{dev}^*\,\eta_- = g, \qquad \mathrm{dev}^*\,\eta_+ = -g.$$

With

$$\mathrm{d}V = \alpha r\,\mathrm{d}r\,\mathrm{d}\varphi\,\mathrm{d}z, \qquad A = -\Delta_{\mathrm{sp}},$$

the normalized scalar equation operator is

$$P_0 = -\mathrm{Box}_g = \partial_t^2 + A.$$

Its retarded and advanced inverses are denoted G_{R} and G_{A}.

The regular host is not globally hyperbolic. For example, a causal diamond large enough to approach $\mathscr{W}$ contains a

sequence with $r$downarrow 0 having no limit in M. The specified spatial realizations, not a global-hyperbolicity theorem applied across an unspecified apex, determine the inverses.

Equation inverses are not action inverses

On symmetric tensors in four dimensions,

$$(\mathrm{P}_{\mathrm{tr}}h)_{ab} = h_{ab} - \frac{1}{2}g_{ab}h^{c}{}_{c}, \qquad \mathrm{P}_{\mathrm{tr}}^{2} = \mathrm{I}.$$

For a declared action Hessian

$$K_{\gamma} = \gamma\,\mathrm{P}_{\mathrm{tr}}P_{0}, \qquad \gamma \neq 0,$$

the corresponding inverse is

$$D_{\gamma}^{\pm} = \gamma^{-1}\mathrm{P}_{\mathrm{tr}}G_{(2)}^{\pm}.$$

Thus a convention $\gamma = -1/4$ gives $D_{\gamma}^{\pm} = -4\mathrm{P}_{\mathrm{tr}}G_{(2)}^{\pm}$, while $\gamma = -1$ gives $-\mathrm{P}_{\mathrm{tr}}G_{(2)}^{\pm}$. These distinct normalizations occur in the supplied X-E-ST and XI-B comparison records [11, 15]. They are not interchangeable.

The observable calculation later uses the explicitly declared Euclidean graviton contraction

$$H^{h}_{ab,cd} = \kappa_{h}(\delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc} - \delta_{ab}\delta_{cd})H$$

with its helicity-resolved interpretation. The numerical formulas are printed at $\kappa_{h} = 1$. This is a normalization of a Gaussian contraction, not an assertion that it equals the inverse of every Hessian . A physical metric perturbation $\delta g = \kappa_{\mathrm{phys}}h$ adds the factor $\kappa_{\mathrm{phys}}^{2}$ to the curvature-square formulas.

Parallel components and helicity

Let $e_{X}, e_{Y}, e_{t}, e_{z}$ be local parallel frames and put

$$e_{\pm} = e_{X} \pm \mathrm{i}e_{Y}, \qquad w = X + \mathrm{i}Y, \qquad \partial_{w} = \frac{1}{2}(\partial_{X} - \mathrm{i}\partial_{Y}).$$

A covariant tensor component contracted with p copies of e_{+} and q copies of e_{-} has weight $\sigma = p - q$.

Proposition (Helicity twist and gluing orientation). *A single-valued tensor has parallel components satisfying*

$$u_\sigma(\varphi + 2\pi) = e^{2\pi i\sigma\alpha} u_\sigma(\varphi).$$

Its holonomy class is $\beta_\sigma = \sigma\alpha \bmod 1$.

The displayed developing map obeys

$$\mathrm{dev}\,(\varphi + 2\pi) = \mathrm{Rot}\,(2\pi\alpha)\mathrm{dev}\,(\varphi) = \mathrm{Rot}\,(-\mu)\mathrm{dev}\,(\varphi).$$

The inverse gluing generator rotates by +μ. These are opposite orientation conventions, not contradictory holonomies.

Proof. Since

$$e_+ = e^{i\theta}(e_r + i e_\varphi^\wedge), \qquad e_- = e^{-i\theta}(e_r - i e_\varphi^\wedge),$$

each transverse contraction contributes the indicated phase. Polar components are periodic. The developing-map identity is immediate.

The geometric and charged-holonomy conventions are those distinguished in I-B-G [5]. No independent Aharonov–Bohm flux is introduced here.

Scalar realizations and helicity channels

The radial operators

Keep an unreduced angular label:

$$u_\sigma(r, \varphi, z) = \sum_{n \in \mathbb{Z}} u_{\sigma n}(r, z) e^{i(n + \sigma\alpha)\varphi}, \qquad a_{\sigma n} = \frac{n}{\alpha} + \sigma, \qquad \nu_{\sigma n} = |a_{\sigma n}|.$$

The developed angular factor is $e^{i a_{\sigma n}\theta}$. Reduction of $\sigma\alpha$ modulo one relabels n; it does not specify the radial realization.

The radial expression on $L^2((0,\infty), r\,\mathrm{d}r)$ is

$$L_\nu = -\partial_r^2 - \frac{1}{r}\partial_r + \frac{\nu^2}{r^2}.$$

For $0 < \nu < 1$, its two scale-invariant self-adjoint realizations are:

$$\mathrm{F}: \quad f(r) = b\, r^\nu + o(r^\nu) \quad \text{in the boundary expansion,}$$

$$\mathrm{E}_0: \quad f(r) = a\, r^{-\nu} + o(r^{-\nu}) \quad \text{with zero regular boundary coefficient.}$$

Both are nonnegative. For $\nu \geq 1$ the endpoint is limit point. For $\nu = 0$, the local basis is $1, \log r$; the only scale-invariant self-adjoint realization is the Friedrichs, no-logarithm one. A ``pure logarithmic branch'' is not a second scale-invariant nonnegative endpoint.

These facts follow from the Bessel boundary form and spectral realizations; a detailed half-line treatment is [19]. The independent role of an apex domain is also emphasized in [20].

Lemma (Resolvent kernels and their finite-rank difference). *For $p > 0$, the radial kernels, relative to $r'\mathrm{d}r'$, are*

$$R_\nu^{\mathrm{F}}(p; r, r') = I_\nu(pr_<)K_\nu(pr_>),$$

$$R_\nu^{0}(p; r, r') = I_{-\nu}(pr_<)K_\nu(pr_>), \qquad 0 < \nu < 1,$$

$$R_\nu^0 - R_\nu^{\mathrm{F}} = \frac{2\sin(\pi\nu)}{\pi} K_\nu(pr) K_\nu(pr').$$

Proof. The Wronskian of I_ν and K_ν fixes the Green jump. The first endpoint uses the chosen boundary solution; the second uses the decaying solution. The identity

$$I_{-\nu} = I_\nu + \frac{2\sin(\pi\nu)}{\pi} K_\nu$$

gives . The Bessel identities are recorded in [21, Chapters 10.27–10.30].

The restricted family

Assumption (Product, channel-diagonal, scale-invariant family). Spatial realizations are direct sums over the helicity and angular channels, followed by the product with $-\partial_z^2$. Boundary parameters are independent of k_z. In

each deficient channel with $0 < \nu < 1$ one chooses either F or E_0; at $\nu = 0$ one chooses the no-logarithm Friedrichs realization. Opposite helicities obey the conjugate reality condition.

No non-diagonal, longitudinally nonlocal, or non-product realization is included in the selection theorem.

A scalar propagation input

The scalar results used from I-B-G are stated explicitly [5, thm:green, thm:speed, prop:E0, thm:transfer]:

1. the nonnegative product realizations above define $\sin(t\sqrt{B})/\sqrt{B}$ and hence source-test retarded and advanced inverses;

2. Friedrichs propagation has intrinsic speed at most one;

3. replacing one deficient Friedrichs channel by E_0 has the retarded correction

$$\Delta G_{\mathrm{R}}(t;x,x') = \frac{2\sin(\pi\nu)}{\pi\alpha} e^{iq(\varphi-\varphi')} \int_0^\infty \cosh(\nu u) F_{\mathrm{R}}(t,\rho(u))\,\mathrm{d}u,$$

where

$$F_{\mathrm{R}}(t,R) = \frac{\Theta(t)}{4\pi R}\delta(t-R), \qquad \rho(u)^2 = r^2 + r'^2 + 2rr'\cosh u + (z-z')^2.$$

In particular the correction starts at

$$\rho_0 = \sqrt{(r+r')^2 + (z-z')^2};$$

4. the scalar kernel agrees with the direct flat kernel on the supported arrival window specified in Section . The correction also follows by taking the Laplace transform in time and the Fourier transform in z, using and the Macdonald product formula. Thus its support statement does not follow from positivity alone.

Gauge operations, source-test domains, and the ladder

The domains actually used

Let $\mathscr{T}_s$ be the compactly supported smooth rank-s tensor sources on M, expressed in parallel frames. Their supports

are bounded away from $r = 0$. A useful intermediate subspace consists of finite angular sums.

All intertwining assertions below mean equality after applying the indicated operations to such sources and pairing the outputs with compact regular tests. In the elliptic proof, one first Fourier transforms in the two longitudinal Euclidean variables; $p > 0$ is then the radial resolvent parameter.

For finite angular sums the proof is an ordinary radial boundary-value calculation. To obtain the source-test kernel identity for general smooth angular sources, take angular partial sums. Fourier coefficients of compact smooth sources decay faster than every power of $|n|$. On compact regular radial sets the Bessel resolvent kernels and fixed derivatives have at most polynomial angular-order bounds after localization. Only finitely many channels are non-Friedrichs. The partial sums therefore converge distributionally, also after the fixed derivatives used below.

This is not a theorem that gauge divergence is bounded on the entire closed quadratic-form domain. The latter remains O6.

Gauge and gauge-fixing maps

For Maxwell,

$$K_1\chi = \nabla\chi, \qquad C_1A = \nabla^a A_a.$$

For symmetric tensors,

$$(K_2\xi)_{ab} = \nabla_a\xi_b + \nabla_b\xi_a, \qquad (C_2h)_b = \nabla^a(\mathrm{P}_{\mathrm{tr}}h)_{ab}.$$

On the regular flat host,

$$C_1K_1 = \mathrm{Box}_g = -P_0, \qquad C_2K_2 = \mathrm{Box}_g = -P_0.$$

The scalar and vector ghost equation expressions are therefore P_0, with their own specified spatial domains.

Definition (Source-test gauge compatibility). A rank-s realization and a rank-$(s-1)$ ghost realization are compatible if their elliptic resolvents satisfy

$$C_sR_s = R_{s-1}C_s$$

on the source-test domain just specified, or equivalently their retarded and advanced equation inverses satisfy

$$C_s G^{\pm}_{(s)} = G^{\pm}_{(s-1)} C_s.$$

The equivalence here is proved by the transformed kernels and uniqueness of their Laplace transforms; it is not an unrestricted commutation claim for an unbounded operator.

Lemma (Corrected Bessel ladder). *For angular number +ν,*

$$\partial_w\left(J_\nu(kr)e^{i\nu\theta}\right) = \frac{k}{2}J_{\nu-1}(kr)e^{i(\nu-1)\theta},$$

$$\partial_{\bar{w}}\left(J_\nu(kr)e^{i\nu\theta}\right) = -\frac{k}{2}J_{\nu+1}(kr)e^{i(\nu+1)\theta},$$

$$\partial_w\left(J_{-\nu}(kr)e^{i\nu\theta}\right) = -\frac{k}{2}J_{1-\nu}(kr)e^{i(\nu-1)\theta},$$

$$\partial_{\bar{w}}\left(J_{-\nu}(kr)e^{i\nu\theta}\right) = \frac{k}{2}J_{-\nu-1}(kr)e^{i(\nu+1)\theta}.$$

For angular number −ν, conjugate the angular factors and interchange $\partial_w, \partial_{\bar{w}}$.

For the decaying solution,

$$\partial_w\left(K_\nu(pr)e^{i\nu\theta}\right) = -\frac{p}{2}K_{\nu-1}(pr)e^{i(\nu-1)\theta}.$$

Proof. Use

$$\partial_w = \frac{1}{2}e^{-i\theta}\left(\partial_r - \frac{i}{r}\partial_\theta\right)$$

and the ordinary and modified Bessel recurrences [21, Sections 10.6 and 10.29]. For $J_{-\nu}$ the angular number remains $+\nu$: one must not replace it by the radial Bessel order when applying the angular derivative.

In particular, lowering a regular channel with $0 < \nu < 1$ in its own angular orientation gives the irregular target $J_{-(1-\nu)}$. Lowering the singular source branch in that orientation gives the regular target $J_{1-\nu}$.

The boundary obstruction

For the displayed convention define

$$\Omega(u,v) = \lim_{r\,\mathrm{downarrow}\,0} r(u\,\partial_r\bar{v} - (\partial_r u)\bar{v}).$$

Then

$$\Omega(r^{-\nu}, r^{\nu}) = 2\nu, \qquad \Omega(r^{\nu}, r^{-\nu}) = -2\nu.$$

Proposition (Channelwise obstruction). *At a fixed longitudinal frequency, a source-test resolvent defect*

$$C_s R_s - R_{s-1} C_s$$

solves the homogeneous target radial equation away from the source and is proportional to its decaying Macdonald solution whenever it is nonzero.

If the target order is less than one, this is a radial rank-one defect between the permitted limit-circle realizations. If the target order is at least one, the differentiated source solution can instead fail to belong to the target Hilbert space near zero; the defect is then understood on regular tests, not as a bounded Hilbert-space rank-one operator.

No finite-rank assertion on the full longitudinal direct integral follows.

Proof. The differential expressions commute on the regular host. Their difference therefore solves the homogeneous target equation. Decay at infinity leaves a multiple of $K_{\nu'}$. The source functional is the boundary-coefficient functional obtained by differentiating the source solution. Its mismatch with the target boundary condition is detected by .

For $\nu' < 1$, $K_{\nu'}$ is locally square integrable and gives the rank statement. For $\nu' \geq 1$, its leading $r^{-\nu'}$ behavior is not in $L^2(r\mathrm{d}r)$. Fourier integration in the longitudinal variables retains an infinite family of such radial fibers.

Maxwell: compatible Friedrichs domain

In a parallel frame the regular Feynman-gauge equation is componentwise $P_0 A = J$. The geometric spin weights are $0, \pm 1$.

Theorem (Maxwell source-test Ward identity). *Use the Friedrichs realization in every vector channel and in*

the scalar ghost channel. Then

$$C_1 G^{\pm}_{(1)} = G^{\pm}_{(0)} C_1.$$

The adjoint source-test gradient intertwining holds as well. Consequently:

1. *a conserved compact source has a zero-past or zero-future response satisfying the Lorenz condition;*

2. *the corresponding field strength satisfies the Maxwell equation with the sign fixed by ;*

3. *smooth finite-energy spectral packets have locally square-integrable field strength near the apex;*

4. *the spectral Feynman-gauge contraction satisfies*

$$\nabla^a_x W^{(1)}_{ab'}(x, x') = -\nabla_{b'} W^{(0)}(x, x').$$

If a deficient transverse channel is changed to E_0 *when* $\alpha > 1/2$*, compatibility fails and generic modes have a field-strength component of order* $r^{-1/\alpha}$*, which is not locally square integrable.*

Proof. The transverse part of divergence is

$$\partial_w A_+ + \partial_{\bar{w}} A_-.$$

For A_+, the developed angular numbers are $a = 1 + n/\alpha$. If $a > 0$, then $a \geq 1$: lowering its regular branch gives a regular scalar branch, including the order-zero endpoint when $a = 1$. If $a < 0$, ∂_w raises the absolute angular order and preserves the regular branch. The mirror statement holds for A_-. Longitudinal derivatives preserve radial order.

Apply these facts to the I_ν endpoint solutions in Lemma . The differentiated solution has the target boundary condition, the transformed inhomogeneous equation, and decay at infinity. Uniqueness of the target resolvent proves the fiber identity. Angular summation and Laplace inversion give the source-test identity. Transposition gives the gradient identity.

For the deficient A_+ channel write $\nu_1 = \alpha^{-1} - 1 \in (0, 1)$. Its singular leading term is

$$A_+ \sim w^{-\nu_1}.$$

Then $\partial_w A_+ \sim r^{-\nu_1 - 1} = r^{-1/\alpha}$. This term occurs in divergence and in the transverse curl. It is not in $L^2(r\mathrm{d}r)$. The regular leading term is $\overline{w}^{\nu_1}$, annihilated by ∂_w; its remaining derivatives have the integrable powers stated in (iii).

The same spectral recurrences give ; equivalently, factor the spectral resolvent identity and integrate its boundary values.

Remark (Residual gauge parameters). Residual parameters are scalar Friedrichs solutions $P_0\chi = 0$ in the stated solution or spectral-packet domain, with gradients in the vector domain. They are not restricted to spacetime-compact homogeneous solutions. Such a restriction would remove the ordinary nontrivial residual solutions rather than describe the causal residual-gauge space.

Spin 2: the Friedrichs defect and curvature-regular branch

The exceptional pair of channels

The symmetric-tensor weights are $0, \pm 1, \pm 2$. The trace reversal commutes with the helicity decomposition.

Theorem (Componentwise Friedrichs obstruction). *Let the tensor realization be componentwise Friedrichs and the vector ghost be the compatible Friedrichs realization of Theorem .*

1. *If* $0 < \alpha \le 1/2$*, the source-test de Donder intertwining identity holds.*

2. *If* $1/2 < \alpha < 1$*, it fails in precisely the two mirror weight-*2 *channels corresponding to unreduced* $(\sigma, n) = (2, -1), (-2, 1)$*. Put*

$$\nu_2 = 2 - \alpha^{-1} \in (0, 1), \qquad \nu_1 = \alpha^{-1} - 1 = 1 - \nu_2.$$

The positive-weight Friedrichs mode has leading term w^{ν_2}*, and*

$$\partial_w w^{\nu_2} \propto w^{-\nu_1}$$

is the irregular target-vector branch. At fixed longitudinal frequency the defect is radial rank one.

3. *The same mode has a linearized curvature component proportional to* $\partial_w^2 w^{\nu_2} \sim r^{-1/\alpha}$*, which is not in* $L^2_{\mathrm{loc}}(r\mathrm{d}r)$.

Proof. In the positive-weight component of $C_2 h$, the term $\partial_w h_{++}$ lowers developed angular number by one. The only positive angular number in $(0, 1)$ among $2 + n/\alpha$ is $2 - \alpha^{-1}$, and this occurs exactly when $\alpha > 1/2$ and $n = -1$. The ladder then gives the forbidden target branch. Every other channel is lowered from order at least one or is raised in absolute order.

When $\alpha = 1/2$, this channel has order zero. Its no-logarithm endpoint solution has a constant leading term, annihilated by the transverse derivative; the next term maps to the regular target. This handles the threshold without treating a constant as a logarithm.

A pure h_{++} component contributes $\partial_w^2 h_{++}$ to the transverse linearized curvature, with a nonzero constant factor. Its leading coefficient is $\nu_2(\nu_2 - 1) \neq 0$. The square has radial integral $\int_0^\varepsilon r^{1-2/\alpha}\mathrm{d}r = \infty$. Proposition proves the rank statement.

The radial coefficient can be displayed without an unspecified Wronskian constant. For the exceptional scalar component, let

$$D_{\nu_2} = \frac{1}{2} e^{-\mathrm{i}\theta}\left(\partial_r + \frac{\nu_2}{r}\right).$$

Using and ,

$$(D_{\nu_2} R^{\mathrm{F}}_{\nu_2} - R^{\mathrm{F}}_{\nu_1} D_{\nu_2}) f = \frac{p \sin(\pi\nu_2)}{\pi} K_{\nu_1}(pr) e^{\mathrm{i}(\nu_2 - 1)\theta} \int_0^\infty K_{\nu_2}(pr') f(r') r' \mathrm{d}r',$$

with the angular channel factored from f. This formula uses unit-normalized angular channel extraction. It proves nonvanishing on suitable compact radial sources.

Definition of the selected branch

Definition (Curvature-regular branch). In the family of Assumption , use Friedrichs in every vector channel. For symmetric tensors use Friedrichs everywhere except, when $\alpha > 1/2$, in $(\sigma, n) = (2, -1), (-2, 1)$, where one uses E_0. At $\alpha = 1/2$ use the no-logarithm realization.

The name *curvature-regular* refers to the regularity criterion below. It is not a physical core-model certification.

Theorem (Restricted selection and compatible de Donder response). *Within Assumption , the curvature-regular branch is the unique spin-1/spin-2 choice satisfying:*

1. *the field-to-ghost source-test identities , with the ghost tower also compatible;*

2. *locally square-integrable field strength for spin 1 and linearized Riemann tensor for spin 2, for smooth spectral packets with arbitrary allowed component amplitudes. Its spatial scalar channel operators are nonnegative. The de Donder residual parameters are vector Friedrichs solutions, with $K_2\xi$ in the selected tensor packet domain.*

This is not a uniqueness theorem among arbitrary non-diagonal or scale-dependent apex realizations.

Proof. Theorem forces the vector choice and hence the gravitational ghost choice. In the exceptional positive tensor channel the singular leading term is

$$h_{++} \sim \overline{w}^{-\nu_2}.$$

It is annihilated by ∂_w and by ∂_w^2. The full Bessel ladder sends $J_{-\nu_2}$ to $J_{1-\nu_2} = J_{\nu_1}$, the regular target branch. This proves compatibility in that channel.

The Bessel series of this singular source has terms

$$\overline{w}^{-\nu_2}\left(1 + c_1 w\overline{w} + c_2 w^2\overline{w}^2 + \cdots\right).$$

Two *w* derivatives kill the first two terms; mixed longitudinal/transverse curvature derivatives are no worse than $r^{1-\nu_2}$, and two longitudinal derivatives are of order $r^{-\nu_2}$. All these powers are locally square integrable because $\nu_2 < 1$.

For the remaining regular channels, the potentially dangerous leading transverse derivatives are either annihilated by the opposite complex derivative or have integrable nonnegative-order descendants. At a zero-order channel the regular Bessel series is in r^2. These observations prove the packet regularity.

Conversely, putting E_0 in any other deficient independent component gives a nonzero field-strength or curvature

derivative of a singular leading power for generic longitudinal packet data. Its order is $r^{-\nu-1}$ or $r^{-\nu-2}$ and is not square integrable. Special pure-gauge combinations do not change this conclusion for a domain allowing arbitrary component amplitudes. Friedrichs in the exceptional tensor channel is excluded by Theorem .

Nonnegativity is the direct sum of the nonnegative Bessel realizations. The residual statement follows from the adjoint intertwiner and .

Remark (What ``finite curvature'' does not mean). The selected potential itself may diverge as $r^{-\nu_2}$. Finite curvature refers to the gauge-invariant derivatives on the specified smooth packet domain, not to componentwise boundedness of h, not to every vector in the closed form domain, and not to a nonlinear distributional curvature of an apex metric.

Ward identity and the free causal interface

Let $W^{(1)}$ be the vector contraction with its scalar normalization, and use the unhalved tensor contraction normalization specified in Section . The compatible identity is

$$\nabla_x^a\Big(W^h_{ab,c'd'} - \frac{1}{2}g_{ab}W^{he}{}_{e,c'd'}\Big) = -\kappa_h\Big(\nabla_{c'}W^{(1)}_{bd'} + \nabla_{d'}W^{(1)}_{bc'}\Big).$$

It follows from the same branch ladders and the trace-reversal normalization. In the flat limit it reduces directly to the derivative of $\kappa_h(\eta_{ac}\eta_{bd} + \eta_{ad}\eta_{bc} - \eta_{ab}\eta_{cd})W$.

Scope boundary (Free causal data, not an interacting differential). The compatible graviton block, Friedrichs vector ghost, and their transposes supply the equation-inverse and intertwining data needed for a cone analogue of a free auxiliary–ghost matrix. This is the appropriate *free-causal interface* comparison with XI-B and X-G-ST [15, 13].

No full conical quantum multiplet algebra, physical cohomology representation, or interacting BRST/BV differential is constructed here. The X-G variational-upgrade obstruction is not discharged. An algebraically written free transformation law does not supply an interacting quantum operation on unconstructed products [14, 16].

Closed kernels and analytic branch labels

Let τ be Euclidean time and define

$$\cosh\eta = \frac{r^2 + r'^2 + (\tau - \tau')^2 + (z - z')^2}{2rr'}, \qquad u = \eta/\alpha, \qquad \psi = \varphi - \varphi'.$$

Put

$$f_b(\zeta) = \frac{e^{-b\zeta}}{1 - e^{-\zeta}}, \qquad S_b(u,\psi) = f_b(u - \mathrm{i}\psi) + f_{1-b}(u + \mathrm{i}\psi).$$

Theorem (Scalar and helicity-resolved kernels). *For $0 \le \beta < 1$, the Friedrichs Euclidean scalar kernel is*

$$W_\beta(x,x') = \frac{S_\beta(u,\psi)}{8\pi^2\alpha rr'\sinh\eta}, \qquad S_\beta(u,\psi) = \sum_{n\in\mathbb{Z}} e^{\mathrm{i}(n+\beta)\psi - |n+\beta|u}.$$

At the compatible spin weights use the unreduced analytic labels

$$b_0 = 0, \qquad b_{+1} = \alpha, \qquad b_{-1} = 1 - \alpha, \qquad b_{+2} = 2\alpha, \qquad b_{-2} = 1 - 2\alpha.$$

If $1 < b < 2$,

$$S_b = S_{b-1} + 2e^{\mathrm{i}(b-1)\psi}\sinh((b-1)u).$$

For $b = 2\alpha > 1$ this changes precisely the exceptional channel to E_0. *Negative-weight kernels are the conjugate counterparts.*

The identification with a nonnegative spatial realization is asserted only for the labels and deficient orders in , not for arbitrary unreduced b.

Proof. Sum the positive and negative angular modes as two geometric series. Their Bessel radial Green integral gives , with the flat normalization at $\alpha = 1, \beta = 0$. For , subtract the two geometric expressions. The changed term replaces $e^{-\nu\eta}$ by $e^{+\nu\eta}$ in the one channel with $\nu = (b-1)/\alpha$. The radial resolvent identity identifies it with the E_0 realization when $0 < \nu < 1$.

A precise tensor assembly

For Euclidean tensor assembly take the orthonormal complex basis

$$u_0 = e_\tau, \qquad u_z = e_z, \qquad u_+ = \frac{e_X - \mathrm{i}e_Y}{\sqrt{2}}, \qquad u_- = \overline{u_+}.$$

The coefficient $u_\pm^\dagger A$ has weight ±1. Let $Q_\sigma^{(s)}$ be the constant orthogonal projector onto total coefficient weight σ in the rank-s tensor representation, restricted to symmetric tensors when $s = 2$. Then

$$W_E^{(1)} = \sum_{\sigma=-1}^{1} Q_\sigma^{(1)} W_{b_\sigma},$$

$$W_E^h = 2\kappa_h \Big(\sum_{\sigma=-2}^{2} Q_\sigma^{(2)} W_{b_\sigma} \Big) \mathrm{P}_{\mathrm{tr}} E.$$

Here $\mathrm{P}_{\mathrm{tr}} E$ is Euclidean trace reversal. The projectors commute with it. For the componentwise Friedrichs tensor kernel replace $b_{\pm 2}$ by their fractional parts. The weight-zero convention is unchanged.

The spectral continuation of these kernels defines the Lorentzian free contractions. Their scalar channel spectra are nonnegative, but the gauge-fixed tensor contraction is indefinite. No positive physical graviton state is inferred from .

Proposition (Threshold analyticity and quotient check). *The compatible kernel labels give an analytic kernel family in α across α = 1/2 at regular Euclidean pairs. The componentwise Friedrichs label reduction does not give the same analytic continuation there.*

At α = 1/N, N ≥ 2, the compatible kernels equal the appropriate finite rotation-character image sums.

Proof. At regular Euclidean pairs, the functions f_b are analytic in b and in positive α. At $b = 1$, $S_1 = S_0$, so the unreduced family passes continuously and analytically through the threshold. The difference is nonzero on one side and changes its derivative at the threshold under the Friedrichs piecewise reduction.

For $\alpha = 1/N$, roots-of-unity projection of the flat angular Fourier series retains exactly the characters $e^{2\pi \mathrm{i}\sigma/N}$. Applying the constant helicity projectors gives the tensor image sum. This is an exact character calculation, not an

inference from finitely many numerical image tests.

Remark (No general core selection). Analyticity and the quotient check are useful consistency properties. Neither proves that a material string core selects this spin-2 branch. A smooth matter core is not a vacuum background for the same linearized Einstein operator; its perturbation problem includes the matter response. This is precisely O2.

Causal support, transposition, and supported transfer

Intrinsic support and the apex-diffracted defect

For spatial points let $\theta = \alpha(\varphi - \varphi')$ on a chosen lift, and put

$$R_k^2 = r^2 + r'^2 - 2rr'\cos(\theta - 2\pi\alpha k) + (z - z')^2.$$

The shortest representative has $|\theta| \le \pi\alpha < \pi$ and gives the intrinsic spatial distance $d(x,x') = R_0$. The corresponding geodesic avoids the apex. Thus the intrinsic causal relation is

$$J_M^{\pm}(t',x') = \{(t,x) : \pm(t - t') \ge d(x,x')\}.$$

Theorem (Causal support and localization). *Both the componentwise Friedrichs and curvature-regular equation inverses have intrinsic finite propagation:*

$$\operatorname{supp} G_{(s)}^{\pm} f \subset J_M^{\pm}(\operatorname{supp} f).$$

Their spin-2 difference is supported in

$$\pm(t - t') \ge \rho_0.$$

The componentwise Friedrichs de Donder defect for $\alpha > 1/2$ is therefore apex diffracted.

Proof. Apply the scalar Friedrichs support theorem to each helicity sector. The only branch corrections are , which begin at ρ_0. Since $\rho_0 > d(x,x')$ for regular points in the shortest angular lift, their support is intrinsic causal. The compatible branch has zero Ward defect, and fixed derivatives do not enlarge support. Applying C_2 to the

branch difference proves the last assertion.

Proposition (Transposition does not require gauge compatibility). *With the natural tensor-density pairing,*

$$\langle f, G_{\mathrm{R}}^{(s)} g\rangle = \langle G_{\mathrm{A}}^{(s)} f, g\rangle$$

for both branches. The same holds for the Hessian-normalized inverses .

The gauge-response identities additionally require the compatible domain; transposition alone does not provide them.

Proof. Each spatial realization is self-adjoint. The sine evolution is symmetric and the retarded and advanced time orderings transpose into one another. Finite tensor sums and trace reversal preserve the identity. The counterexample shows why this argument cannot also prove gauge compatibility.

The arrival window and retained shadow

An angularly small chart lies in

$$r > r_U > 0, \qquad \varphi \in I, \qquad |I| < \pi,$$

with a fixed developing lift. Define

$$T_{\mathrm{arr}}(x, x') = \min\big(\{\rho_0\} \cup \{R_k : k \neq 0,\ |\theta - 2\pi\alpha k| \le \pi\}\big).$$

At an image boundary the included term has length ρ_0. A sufficient uniform lower bound is

$$T_{\mathrm{arr}} \ge 2 r_U \sin(\pi\alpha/2) > 0.$$

Theorem (Supported Green-operator replacement). *Let f be a compact smooth tensor source in an angularly small chart and let $\mathcal{O}$ be a regular observation region in that chart. Assume, for the required time direction,*

$$\pm(t - t') < T_{\mathrm{arr}}(x, x') \quad \text{for every} (t, x) \in \mathcal{O}, ?(t', x') \in \operatorname{supp} f.$$

Then, in the parallel frame, the intrinsic spin-1 and spin-2 responses of either branch equal the corresponding flat equation responses on $\mathcal{O}$. The same statement holds for their scalar or vector ghosts.

To insert this replacement into a supported companion calculation, its complete causal shadow must also lie in the retained regular laboratory. The arrival inequality does not supply that independent support condition.

Proof. In each scalar helicity sector the direct developed geometric contribution has unit coefficient. Additional images begin at the lengths in , and diffraction begins at ρ_0. Thus the difference kernel vanishes on the open pair region . The E_0 corrections also begin at ρ_0. The projectors are constant in the parallel frame, so componentwise replacement gives the tensor result. This is the scalar supported-transfer theorem of I-B-G applied to the explicitly specified sectors [5, thm:transfer].

A later composition may use intermediate sources and additional causal dependence. Their containment is a separate hypothesis, as in the supported ST contracts [9, 10].

Scope boundary (Green equality is not covariance equality). Equality in concerns retarded or advanced operations. It does not identify the symmetric covariance, spectral ground state, or Hadamard remainder of the flat completion.

For example, the regular scalar Wick square on the cone can be nonzero in an arbitrarily small regular chart even though its sufficiently short causal inverse agrees there with the direct flat one [7]. Ground-state selection is global spectral data, not a consequence of short-time causal agreement.

Bernoulli expansion and finite coincidence jets

Let

$$\Theta = \theta - \theta', \qquad \zeta = \eta - \mathrm{i}\Theta, \qquad \bar{\zeta} = \eta + \mathrm{i}\Theta,$$

where the bar denotes the displayed algebraic counterpart in the Euclidean calculation. Define

$$H_b = W_b - W_{\mathrm{flat}} \circ \mathrm{dev}.$$

Lemma (Convergent Bernoulli regular part). *Near every regular diagonal point,*

$$H_b = \frac{\eta/\sinh\eta}{8\pi^2 r r'} \sum_{k \geq 2} a_k(b) P_k(\eta, \Theta),$$

where

$$a_k(b) = \frac{B_k(b)\alpha^{-k} - B_k(0)}{k!}, \qquad P_k = \frac{(-1)^k\zeta^{k-1} + \bar{\zeta}^{k-1}}{\eta}.$$

The series converges locally for $|\eta \pm \mathrm{i}\Theta| < 2\pi\alpha$. *The regular representative is real analytic in local Cartesian variables, with complex values allowed for a single helicity.*

The first polynomials are

$$P_2 = 2,$$

$$P_3 = 4\mathrm{i}\Theta,$$

$$P_4 = 2\eta^2 - 6\Theta^2,$$

$$P_5 = 8\mathrm{i}\Theta(\eta^2 - \Theta^2),$$

$$P_6 = 2\eta^4 - 20\eta^2\Theta^2 + 10\Theta^4.$$

A total derivative jet of order d requires $k \le d + 2$. *In particular, Maxwell quadratic field strengths use* $k \le 4$, *while curvature squares use* $k \le 6$.

Proof. Apply

$$\frac{te^{bt}}{e^t - 1} = \sum_{k=0}^{\infty} B_k(b)\frac{t^k}{k!}$$

to the two terms of S_b, using $B_k(1 - b) = (-1)^k B_k(b)$ [21, Section 24.2]. The pole terms give the flat direct kernel and the constant terms cancel. Subtracting the flat expression gives .

Each numerator defining P_k is divisible by η. It is a homogeneous polynomial of degree $k - 2$ in η, Θ, even in η. The quantity η^2 is analytic near coincidence, as is Θ in a regular angular chart. Hence the apparent η denominator creates no singularity. The degree proves the jet cutoff.

Remark (Scalar normalization and complex first jets). At coincidence gives

$$H_\beta(x,x) = \frac{\alpha^{-2}[1-6\beta(1-\beta)]-1}{48\pi^2 r^2},$$

the scalar Wick-square coefficient of I-B-VP. Its second jets supply the regular scalar stress coefficients. The odd, complex first jet must be retained for a charged current; taking a real part before that jet would erase the current information [7, 8]. These are normalization comparisons, not transfers of a scalar contact classification to spin 1 or spin 2.

The exact finite polynomial needed for all computations is specified in Appendix . It replaces a proof that merely says that an unavailable script performed the contractions.

Regular local observables

Definitions and normalization

The bracket $\langle\cdot\rangle_{\mathrm{ren}}$ below means flat Hadamard subtraction in a developing parallel chart and coincidence of the resulting smooth regular kernel. The observables are defined on M, not on $\mathscr{W}$. Set $\hbar = 1$; it can be restored as an overall factor.

For Maxwell use Heaviside–Lorentz normalization,

$$T_{ab} = F_{ac}F_b{}^c - \frac{1}{4}g_{ab}F_{cd}F^{cd}.$$

For the symmetric-tensor calculation use with $\kappa_h = 1$ and the linearized Riemann operator

$$(\mathcal{R}h)_{abcd} = \frac{1}{2}\big(\partial_c\partial_b h_{ad} + \partial_d\partial_a h_{bc} - \partial_d\partial_b h_{ac} - \partial_c\partial_a h_{bd}\big).$$

Its overall sign is immaterial to the quadratic results, but is fixed here.

Define

$$E_{ij} = (\mathcal{R}h)_{titj}, \qquad B_{ij} = \frac{1}{2}\varepsilon_{ikl}(\mathcal{R}h)_{kltj}.$$

The compatible Ricci–Riemann identity below identifies these with the corresponding linearized Weyl quantities

at the level of the stated regular contractions.

Lemma (Finite contraction identities). *Put $q = \alpha^{-1}$. Applying the definitions above to and , using the jets in Lemma , gives*

$$8\pi^2 r^4 \langle E^2_E \rangle_{\mathrm{ren}} = \frac{q^4 + 10q^2 - 11}{90}, \qquad \langle B^2_E \rangle_{\mathrm{ren}} = -\langle E^2_E \rangle_{\mathrm{ren}},$$

and

$$8\pi^2 r^6 \langle E^E_{ij} E^E_{ij} \rangle_{\mathrm{ren}} = \frac{2q^6 + 21q^4 + 168q^2 - 191}{1260},$$

$$\langle B^E_{ij} B^E_{ij} \rangle_{\mathrm{ren}} = -\langle E^E_{ij} E^E_{ij} \rangle_{\mathrm{ren}}.$$

The labels E on the fields in these formulas denote Euclidean time components.

Proof. Both statements are finite polynomial identities. For total degree two, insert P_2, P_3, P_4 in and apply $F_{ab} = \partial_a A_b - \partial_b A_a$ at both endpoints. For total degree four, insert through P_6 and apply twice. The constant helicity projectors prescribe every tensor coefficient.

For an explicit coefficient-level verification, use in Appendix , the displayed Bernoulli polynomials there, and the finite component contraction rules . The resulting numerators are respectively $q^4 + 10q^2 - 11$ and $2q^6 + 21q^4 + 168q^2 - 191$. Their common flat value at $q = 1$ is zero. No numerical limit or image interpolation is involved.

Maxwell stress

Theorem (Maxwell regular one-point functions). *For the compatible Friedrichs Maxwell contraction,*

$$\langle T^{\mu}{}_{\nu} \rangle_{\mathrm{ren}} = \frac{(\alpha^{-2} - 1)(\alpha^{-2} + 11)}{720\pi^2 r^4} \mathrm{diag}\,(1, 1, -3, 1)$$

in the order (t, r, φ, z), and

$$\langle F_{\mu\nu} F^{\mu\nu} \rangle_{\mathrm{ren}} = 0,$$

$$\langle E^2\rangle_{\mathrm{ren}} = \langle B^2\rangle_{\mathrm{ren}} = -\frac{(\alpha^{-2}-1)(\alpha^{-2}+11)}{720\pi^2 r^4}.$$

The stress is conserved and traceless on M. Its static energy density is $\rho = -\langle T^t{}_t\rangle_{\mathrm{ren}} < 0$.

Proof. Wick rotation gives $\langle E^2\rangle_L = -\langle E_E^2\rangle$ and $\langle B^2\rangle_L = \langle B_E^2\rangle$. Equation gives the field squares and $F^2 = 2(B^2 - E^2) = 0$. The energy density is $(E^2 + B^2)/2$.

Stationarity, longitudinal boosts, rotation covariance, scaling, and reflection symmetry give a mixed tensor $r^{-4}\mathrm{diag}\,(a, b, c, a)$. Radial conservation gives $c = -3b$. Tracelessness gives $a = b$. The computed energy fixes their common coefficient.

This agrees with the boundary-free electromagnetic result printed in [22, Eq. (38)]. That source uses Gaussian field-square conventions; its E^2 and B^2 are 4π times the Heaviside–Lorentz quantities used here.

Scope boundary (No Maxwell sourcing inference). Equation is a regular one-point expectation in the declared free prescription. Its symmetry, conservation, and trace do not imply that it sources the prescribed cone. No semiclassical Einstein equation or backreaction solution has been imposed [3, prop:not-source] [9, prop:noeinstein].

Curvature squares and the Ricci diagnostic

Proposition (Ricci–Riemann identity). *For the compatible contraction, the regular correlator*

$$\langle (\mathrm{Ric}\, h)_{ab}(x)(\mathcal{R}h)_{c'd'e'f'}(x')\rangle$$

vanishes at regular separated points, and hence its flat-subtracted coincidence limit vanishes for every component.

This is a consequence of the Ward identity and the bisolution equation. Vanishing of a finite selection of coincidence components is not, conversely, a proof of the full Ward identity.

Proof. On the regular flat host,

$$(\mathrm{Ric}\, h)_{ab} = \frac{1}{2}\big(\partial_a (C_2 h)_b + \partial_b (C_2 h)_a - \mathrm{Box}_g\, h_{ab}\big).$$

The wave term annihilates the first endpoint of the two-point bisolution. By , the remaining terms are pure gauge derivatives in the primed tensor slot. The primed linearized Riemann operator annihilates those derivatives. The direct flat contraction obeys the same identity. Its subtraction therefore preserves zero, and the analytic regular remainder permits coincidence.

Theorem (Curvature-regular graviton contractions). *For* $\kappa_h = 1$,

$$\langle E_{ij} E^{ij} \rangle_{\mathrm{ren}} = \langle B_{ij} B^{ij} \rangle_{\mathrm{ren}} = \frac{(\alpha^{-2} - 1)(2\alpha^{-4} + 23\alpha^{-2} + 191)}{10080\pi^2 r^6},$$

$$\langle \mathscr{T}_{tttt} \rangle_{\mathrm{ren}} = \langle E_{ij} E^{ij} + B_{ij} B^{ij} \rangle_{\mathrm{ren}} = \frac{(\alpha^{-2} - 1)(2\alpha^{-4} + 23\alpha^{-2} + 191)}{5040\pi^2 r^6}.$$

Here $\mathscr{T}_{tttt}$ *denotes the stated Bel–Robinson component in this normalization. Moreover,*

$$\langle (\mathcal{R}h)_{abcd} (\mathcal{R}h)^{abcd} \rangle_{\mathrm{ren}} = 0.$$

The linearized curvature polynomial is invariant under $h \mapsto h + K_2 \xi$ *on the regular flat host.*

No complete Bel–Robinson tensor or its conservation theorem is asserted.

Proof. Two Lorentzian time indices multiply E^E by a minus sign, which cancels in its square; one time index multiplies B^E by a factor whose square is minus one. Thus gives equality of the Lorentzian electric and magnetic squares. Factor

$$2q^6 + 21q^4 + 168q^2 - 191 = (q^2 - 1)(2q^4 + 23q^2 + 191)$$

to obtain . Their sum gives .

Proposition removes the Ricci terms in the contracted curvature decomposition. The four-dimensional Weyl identity gives $C_{abcd} C^{abcd} = 8(E_{ij} E^{ij} - B_{ij} B^{ij})$, whose expectation is zero. Finally, commuting ordinary derivatives in gives $\mathcal{R}(K_2 \xi) = 0$.

Remark (Gauge invariance versus a physical state). The last algebraic gauge invariance is true for the

linearized curvature polynomial regardless of an apex branch. The compatible Ward identity is what additionally makes the chosen contraction consistent with the specified gauge/ghost interface. It does not by itself establish positivity on physical cohomology or identify an action normalization.

Proposition (Friedrichs coincidence diagnostics). *For* $1/2 < \alpha < 1$, *replacing only the exceptional tensor labels by their Friedrichs reductions gives the Euclidean regular contractions*

$$\langle (\mathcal{R}h)_{abcd} (\mathcal{R}h)_{abcd} \rangle_{\mathrm{ren}} = \frac{2(\alpha-1)(2\alpha-1)(62\alpha^2-17\alpha+1)}{15\pi^2\alpha^6 r^6},$$

$$\langle (\mathrm{Ric}\, h)_{XX} (\mathcal{R}h)'_{XYXY} \rangle_{\mathrm{ren}} = \frac{(\alpha^2-1)(2\alpha-1)(13\alpha-1)}{60\pi^2\alpha^6 r^6}.$$

They are nonzero throughout that open interval and tend to zero at $\alpha = 1/2$.

Proof. Use the same degree-four polynomial , with $b_{+2} = 2\alpha - 1$ and $b_{-2} = 2 - 2\alpha$. Subtracting the compatible degree-four contractions leaves only the two flip terms . Applying and the displayed Ricci operator gives –. All factors besides $(\alpha - 1)$ or $(\alpha^2 - 1)$ are positive for $1/2 < \alpha < 1$.

The nonzero Ricci–Riemann quantity is a coincidence diagnostic of the boundary incompatibility already proved independently by . It is not used to infer that obstruction from a numerical sample.

Scope boundary (No gravitational sourcing inference). The curvature squares and the Bel–Robinson component are Gaussian free-field contractions in a prescribed normalization. They are not a gravitational stress tensor derived from a renormalized effective action, and are not a source law for the cone. No nonlinear Einstein dynamics, matter-core evolution, or backreaction follows [3, 9].

The exact spectral polynomial identity

For $s \in \mathbb{N}$ define

$$I_s(q) = \int_0^\infty \frac{\omega \prod_{j=1}^{s} (\omega^2 + j^2)}{e^{2\pi\omega/q} - 1} \, \mathrm{d}\omega.$$

Expansion of the finite product and

$$\int_0^\infty \frac{\omega^m}{e^{a\omega}-1}\mathrm{d}\omega = \frac{\Gamma(m+1)\zeta(m+1)}{a^{m+1}}, \qquad m>0,$$

gives

$$240[I_1(q)-I_1(1)] = (q^2-1)(q^2+11),$$

$$1008[I_2(q)-I_2(1)] = (q^2-1)(2q^4+23q^2+191).$$

Consequently

$$\rho_{\mathrm{EM}} = -\frac{I_1(q)-I_1(1)}{3\pi^2 r^4}, \qquad \langle E_{ij}E^{ij}\rangle_{\mathrm{ren}} = \frac{I_2(q)-I_2(1)}{10\pi^2 r^6}.$$

These are algebraic integral identities. They do not supply a thermal interpretation of the state. Rindler field calculations provide historical comparison [28].

Evidence, retained checks, and the O5 protocol

Evidence classes

Following the reporting discipline compared with XVI-O1-ST [17], distinguish:

\texttt{ANALYTIC} A statement established by the written calculation with its explicitly identified analytical inputs.

\texttt{SUPPLIED} A computational value reported in the supplied predecessor, not independently rerun in this revision.

\texttt{UNCERTIFIED} A numerical agreement or enclosure claim without a complete total-error certificate.

The predecessor names scripts, logs, a checkpoint file, and hashes. Those artifacts were not supplied as executable materials here. Their names and reported values are retained below as provenance, not as independently inspected files. No detached execution or high-precision run is newly asserted.

Retained numerical reports

begingroup toprule Reported check Supplied result Supplied artifact name

midrule endfirsthead toprule Reported check Supplied result Supplied artifact name

midrule endhead bottomrule endfoot

Scalar jets versus I-B-VP Exact symbolic residuals reported as zero test_scalar.py

Vector/tensor rotation images, $N = 2, \ldots, 6$ Maximum errors reported as 7.4×10^{-32} and 1.2×10^{-31} image_check.json

Maxwell coefficient, symbolic Ratio to reported as one; $F^2 = 0$ em_T.json

Maxwell full kernel, $\alpha = 1/\sqrt{2}$ 0.0018294102602088764290 versus the reported closed-form value ending 428848; relative difference 8.7×10^{-20} em_numeric_0.707107.json

Maxwell full kernel, $\alpha = 1/\pi$ 0.026048710251421726572 versus the reported closed-form value ending 573544; relative difference 4.2×10^{-20} em_numeric_0.31831.json

Spin-1 compatible Ward tests Relative residual at most 1.8×10^{-31} at $\alpha = 1/\sqrt{2}, \sqrt{2} - 1$ ward__geom.json

Spin-1 singular transverse test Reported relative residuals 7.6×10^{-2} and 4.1×10^{-3}: failure retained ward_0.7071_e0vec.json

Spin-2 Friedrichs Ward test Residual 8.6×10^{-3} against right side of size 0.58: failure retained ward_0.7071_fried.json

Spin-2 compatible Ward test Residual at most 1.3×10^{-31} ward_0.7071_geom.json

Spin-2 test below the threshold At $\alpha = \sqrt{2} - 1$, both branches coincide; reported residual at most 1.3×10^{-31} ward_0.4142_.json

Unreduced-label scalar four-jets Reported numbers 6.75, 2.97886011720255, 2.71257820674942, agreement

about 10^{-15} jetcheck_b2.json

Graviton image checks At $\alpha = 1/2, 1/3, 1/4$, reported values $3/4, 32/9, 51/4$ in units $1/(8\pi^2 r^6)$ image_spin2.json

Ricci–Riemann checks Zero reported for the compatible branch on the tested components; 12 nonzero entries reported for Friedrichs spin2_.json

endgroup

All entries in this table have status SUPPLIED. Numerical entries additionally have status UNCERTIFIED. No raw unrounded pair is reconstructed from an abbreviated decimal.

The supplied Ward tests used the Euclidean pair

$$x = (0.1, 1, 0.2, 0.05), \qquad x' = (0.35, 0.8, 0.45, 0.2)$$

in (τ, X, Y, z). The predecessor discarded an attempted vector singular extension at $\alpha < 1/2$ because it was not an admissible self-adjoint channel choice. That exclusion is retained.

Finite checks do not cover arbitrary angles

Image checks at $\alpha = 1/N$ do not by themselves determine a function at irrational α, or in the interval $1/2 < \alpha < 1$. In particular, those image points do not supply a numerical test of the non-Friedrichs spin-2 channel in that interval.

The analytical quotient identity in Proposition and the finite contraction proof in Lemma are separate from the numerical tests. This is the class-relative coverage distinction compared with Oai282-A [18].

Proposed independent O5 protocol

The following protocol is a proposal, not an execution report. Its target is a full-kernel check of at irrational $\alpha > 1/2$.

1. Freeze exact inputs, for example $\alpha = 1/\sqrt{2}$ and $\alpha = \sqrt{3}/2$, the regular external point, κ_h, the Lorentzian/Euclidean index convention, and the field-strength differential operators.

2. Evaluate the full kernels and , subtracting the direct flat kernel before taking the high-order coincidence limit. The $b = 2\alpha > 1$ term must remain unreduced.

3. Use a differentiation method independent of the degree-four polynomial used for the prediction. Possible choices are a sufficiently high-order symmetric difference with a proved remainder, or a complex-contour derivative with a specified holomorphic domain and quadrature bound.

4. Repeat with an independent mode or resolvent representation, using for the exceptional channels. Record angular truncation, spectral cutoff, low-energy treatment, and every tail estimate.

5. Retain, before rounding: both paired values, their difference, absolute and relative normalizations, working precision, differentiation step or contour, quadrature tolerances, truncations, and software versions.

6. Separate the error ledger:

$$\varepsilon_{\text{total}} \le \varepsilon_{\text{input}} + \varepsilon_{\text{subtraction}} + \varepsilon_{\text{derivative}} + \varepsilon_{\text{quadrature}} + \varepsilon_{\text{tail}} + \varepsilon_{\text{roundoff}}.$$

A tolerance requested from an integrator is not itself a proved value of $\varepsilon_{\text{quadrature}}$.

7. Preserve failed comparisons and branch/sign variants. A regression expected to fail remains a failure diagnostic, not an error to delete.

8. Label an ordinary high-precision residual UNCERTIFIED unless the full error bound is justified. Even a certified numerical comparison verifies only the declared tests; it is not the proof of the all-α formula.

O5 remains open until such same-target artifacts and comparisons are supplied.

Literature comparison and novelty boundary

The modern comparison literature already contains cosmic-string photon and graviton Green functions, Ward identities, and vacuum averages [23, 24, 26]. No priority is claimed for these subjects.

For clarity, literature access and mathematical use are graded separately:

$*$ A specific displayed formula is available and used for the stated comparison.

$\ddagger$ A primary bibliographic or abstract record is confirmed; no uninspected equation is attributed as verified.

$\dagger$ A historical comparison is retained from the supplied record without a new full-text audit.

$*$ Bezerra de Mello–Bezerra–Saharian [22, Eq. (38)] prints the boundary-free Maxwell energy $-(q^2-1)(q^2+11)/(720\pi^2 r^4)$. It matches after signature and field-square units are identified. No shell contribution is included here.

$\ddagger$ Allen–McLaughlin–Ottewill [26] explicitly emphasize Ward identities and smoothing of the curvature singularity. This is consistent with the need to distinguish the local equation from its apex domain. The present paper does not identify their rounded-core construction with either idealized realization.

$\ddagger$ Dowker [24] treats arbitrary spin and Bel–Robinson averages. A coefficient-level comparison with , including the normalization of h, remains O1. The existence of the earlier calculation precludes presenting the general observable as newly introduced here.

$\ddagger$ Linet and Dowker supply classical cone-kernel context [27, 25]. Kay–Studer and Dereziński–Richard supply the boundary-domain context [20, 19].

$\dagger$ The Rindler comparison in the supplied record refers to Candelas–Deutsch [28]. The spectral equalities in this paper are independently evaluated algebraic identities and do not import a Rindler-state identification.

The narrower contribution claimed here is the explicit helicity-domain audit for the declared companion problem: the exceptional channels, their radial obstruction, the restricted mathematical selection, the normalization ledger, and the exact supported/non-supported interface. The physical core interpretation and completed quantum gauge theory remain separate.

Consequences for the companion interfaces

The following table records proposed analytical handoff information. It does not assert that any external

manuscript or registry has been changed. Source labels refer to the supplied comparison records; an unavailable full proof is not treated as independently audited.

begingroup toprule Record Named interface Supply and non-transfer

midrule endfirsthead toprule Record Named interface Supply and non-transfer

midrule endhead bottomrule endfoot

I-B; En09-A Finite destructive instrument; Einstein comparator The classical code and residual do not acquire the Green operators or the tensors here. No continuum source or geometry update is inferred.

I-B-G Scalar inverses, helicity twists, transfer Spin-1/spin-2 Green operators and restricted curvature-regular selection are supplied here in the source-test sense; no interacting product or dynamics. The scalar analytical inputs are stated explicitly.

I-B-VP family Scalar state, Wick square, stress, current; V5 operator limit Scalar normalization and complex-first-jet conventions are comparison controls. Scalar apex contacts are not spin-2 boundary parameters. The scalar core limit does not select a gravitational matter-core limit.

X-E-ST thm:free; action inverse The equation inverse is available intrinsically. Its action normalization must be supplied by –. No coefficient −4 is identified with −1.

X-F-ST Transposition; response covariance Transposition holds for both branches. The compatible branch supplies the necessary linear gauge-response intertwining. It does not transfer a nonlinear background-gauge covariance theorem, a changed-background apex domain, or a complete tree construction.

X-G-ST Auxiliary–ghost causal matrix; free BRST language Compatible graviton/vector-ghost intertwiners are available, and the Friedrichs graviton obstruction is apex diffracted. A full conical free algebra and an interacting quantum differential are not constructed. X-G's variational-upgrade obstruction is untouched.

X-H-ST Off-shell algebraic identities No additional off-shell or BV conclusion follows from a choice of spectral boundary domain. No interaction-level result is transferred.

X-I-ST Non-global-hyperbolicity; free causal interface The missing apex condition is supplied as an explicit input, not removed by a global-hyperbolicity claim. No physical cohomology representation follows.

X-J-ST; X-K-ST Charged or documentary comparisons A compatible free photon inverse is available. No charged interacting coefficient or further analytic interface is constructed here.

X-L-ST Ghost support and residual language Scalar Friedrichs ghosts for Maxwell and vector Friedrichs ghosts for gravity are available. Spacetime-compact homogeneous ghosts are not substituted for the residual solution space. Width is not core thickness or a loop regulator.

XI-B lem:Green; thm:graviton Causal inverse and residual-gauge terminology are used only after the normalization dictionary. The flat-completion algebra is not imported. The full auxiliary–ghost quantum construction remains an additional task.

XVI-O1-ST Integrity and residual retention The numerical protocol retains raw values and distinct error sources. No unrun computation or missing artifact is relabeled as a certificate.

Oai282-A Finite-test coverage Finite image and Ward tests are not exhaustive continuum coverage without the stated analytical identities.

endgroup

The table is not a proof transfer. In particular, changing a background metric can change its boundary domains. The source-test intertwiners do not by themselves prove naturality of an entire extension family under arbitrary background changes.

Open items, debugging ledger, and release

The six original open items

Their statuses are unchanged.

1. Literature coefficient comparison: open. Match with Dowker's Bel–Robinson coefficient and compare the

graviton construction with Allen–McLaughlin–Ottewill. The missing data are the same field normalization, branch, subtraction, and observable convention.

2. Physical spin-2 core selection: open. A matter core, its coupled non-vacuum linearization, a transport between the core and cone Hilbert spaces, and a controlled limit of the relevant operators and states are required. An idealized branch criterion is not that limit.

3. Full Bel–Robinson tensor: open. Only the displayed *tttt* component and the Ricci diagnostic are released. A complete tensor calculation and its distributional or regular conservation statement are not supplied.

4. Non-diagonal extensions and spinors: open. The selection theorem is restricted to Assumption . Spin-1/2, spin-3/2, and non-diagonal gauge-domain classifications require their own operator and ghost analysis.

5. Independent full-kernel irrational-angle graviton check: open. Section specifies the missing numerical comparison. Existing image sums and shared-jet tests are not a completed independent check.

6. Full form-domain obstruction theorem: open. The present proofs are channelwise and on compact source-test or smooth packet domains. A theorem about closed domains, graph norms, and an unbounded gauge map on the full form space requires estimates beyond these calculations. Radial rank one is not full operator rank one.

Debugging corrections

toprule Potential overstatement or error Correction in this revision

midrule endfirsthead toprule Potential overstatement or error Correction in this revision

midrule endhead bottomrule endfoot

The developing map itself glues by $+\mu$ It advances by $2\pi\alpha = -\mu$ modulo 2π. The inverse generator is the $+\mu$ convention.

The angular number equals the signed Bessel order on an irregular branch They differ. Equation keeps the angular number fixed while differentiating $J_{-\nu}$.

$\Omega(r^{-\nu}, r^{\nu}) = -2\nu$ for the printed boundary form For that form it is $+2\nu$, as in .

A logarithmic zero channel is another nonnegative scale-invariant endpoint At order zero, only the no-logarithm realization is scale invariant.

An unreduced twist means a different holonomy The holonomy is a class modulo one. The unreduced label records a radial branch in the closed kernel.

A radial rank-one statement is a full product finite-rank theorem The statement is fiberwise in longitudinal frequency. The full form-domain theorem remains O6.

A scalar spectral theorem automatically commutes an unbounded divergence through every function The source-test result is proved by radial boundary matching and Laplace-transform uniqueness.

Every vector in the selected form domain has finite linearized curvature Only the stated smooth spectral-packet criterion is proved; no such full-domain assertion is made.

The action inverse and contraction normalization are the same input Equations – and keep them separate.

A total $2m$ jet uses only B_k with $k \le m + 2$ A total order-d jet needs $k \le d + 2$. Curvature squares need through B_6.

Maxwell stress requires four derivatives of the potential covariance It requires one derivative at each field-strength endpoint, total order two.

A subset of zero Ricci–Riemann components proves the Ward identity The Ward identity is proved independently. It implies the complete Ricci–Riemann identity.

Compatible Green operations transfer the symmetric flat state Supported causal equality does not transfer covariance or ground-state selection.

Self-adjointness proves gauge-response covariance It proves transposition. Domain-compatible intertwiners are additional.

The reported scripts and logs were rerun here They are retained as SUPPLIED; numerical reports are

UNCERTIFIED without the complete error artifacts.

Mathematical release

Theorem (I-B-G-S release in the stated domain). *For the prescribed host, the product channel family, the source-test gauge operations, and the contraction normalizations declared here:*

1. *the helicity twists and channel realizations define intrinsic free equation inverses;*

2. *the Maxwell Friedrichs realization has the scalar Friedrichs source-test Ward interface;*

3. *the componentwise Friedrichs spin-2 realization has the explicit exceptional-channel defect for $\mu < \pi$;*

4. *within the restricted scale-invariant family, the curvature-regular branch restores source-test intertwining and the stated packet curvature regularity;*

5. *the causal branch difference is apex diffracted, both branches transpose, and the supported arrival-window replacement holds without a covariance-transfer claim;*

6. *the finite Bernoulli-jet prescription gives , , , and the compatible Ricci–Riemann identity. The six open items remain open. No interacting quantum differential, full conical gauge algebra, physical core selection, backreaction, or spacetime emergence is released.*

Proof. Items (i)–(iv) are Proposition , Lemma , Theorems , , and , with the explicitly stated scalar propagation input. Item (v) is Theorem , Proposition , and Theorem . Item (vi) is Lemmas , , Theorems , , and Proposition . None supplies the additional domains, complete tensors, core model, or interacting products listed in O1–O6.

The exact standing boundary remains:

boxed free, extension-dependent conical Green data: supplied,
interacting products and dynamics: not constructed,
O.1 on the cone = texttt *NOT_CLOSED*,
XVI-N not discharged; XV-B not opened,
texttt *bulk_symmetry_restored* = texttt *false*.

These are scope-qualified programme statements, not an independent audit of every differently scoped companion version.

appendix

Finite algebraic prescription for the observable jets

This appendix specifies the finite calculation in Lemma . It has no code dependency.

By translation, rotation, and scaling, evaluate at the Euclidean base point

$$x_0 = (\tau, X, Y, z) = (0, 1, 0, 0).$$

Use endpoint increments

$$x = (u_\tau, 1 + u_X, u_Y, u_z), \qquad x' = (v_\tau, 1 + v_X, v_Y, v_z),$$

and discard total degree above four after every operation. Set

$$r = \sqrt{(1 + u_X)^2 + u_Y^2}, \qquad r' = \sqrt{(1 + v_X)^2 + v_Y^2},$$

$$\Theta = \arctan \frac{u_Y}{1 + u_X} - \arctan \frac{v_Y}{1 + v_X},$$

$$q_* = \frac{(r - r')^2 + (u_\tau - v_\tau)^2 + (u_z - v_z)^2}{2rr'}.$$

Since q_* begins in degree two,

$$V := \eta^2 = 2q_* - \frac{1}{3}q_*^2 + O(\text{degree6}).$$

Write $a_k = a_k(b)$. The complete degree-four scalar regular jet is

$$\begin{aligned} \mathcal{J}_b(u, v) = \frac{1}{8\pi^2}\Big[\frac{1}{rr'}\Big(1 - \frac{V}{6} + \frac{7V^2}{360}\Big)[\quad & 2a_2 + 4ia_3\Theta + a_4(2V - 6\Theta^2) \\ & + 8ia_5\Theta(V - \Theta^2) \\ & + a_6(2V^2 - 20V\Theta^2 + 10\Theta^4)]\Big]_{\le 4}. \end{aligned}$$

For a degree-two calculation truncate this expression at total degree two.

The Bernoulli polynomials required are

$$B_2(b) = b^2 - b + \frac{1}{6},$$

$$B_3(b) = b^3 - \tfrac{3}{2}b^2 + \tfrac{1}{2}b,$$

$$B_4(b) = b^4 - 2b^3 + b^2 - \tfrac{1}{30},$$

$$B_5(b) = b^5 - \tfrac{5}{2}b^4 + \tfrac{5}{3}b^3 - \tfrac{1}{6}b,$$

$$B_6(b) = b^6 - 3b^5 + \tfrac{5}{2}b^4 - \tfrac{1}{2}b^2 + \tfrac{1}{42}.$$

Every jet is an ordinary coefficient:

$$\left[\partial_x^{\gamma}\partial_{x'}^{\delta}H_b\right] = \gamma!\,\delta!\,[u^{\gamma}v^{\delta}]\mathscr{J}_b, \qquad |\gamma| + |\delta| \le 4.$$

The overall r^{-4} or r^{-6} is restored by scaling.

For a fully explicit component assembly, let

$$M_{ab} = \sum_{\sigma=-1}^{1} (Q_\sigma^{(1)})_{ab}\mathscr{J}_{b_\sigma}, \qquad N_{ab,cd} = 2\sum_{\sigma=-2}^{2} (Q_\sigma^{(2)}\mathrm{P}_{\mathrm{tr}}E)_{ab,cd}\mathscr{J}_{b_\sigma}.$$

All indices run over τ, X, Y, z. Use the finite rules

$$\mathscr{F}_{ab,cd} = \partial_a\partial_{c'}M_{bd} - \partial_a\partial_{d'}M_{bc} - \partial_b\partial_{c'}M_{ad} + \partial_b\partial_{d'}M_{ac},$$

$$\mathscr{R}_{abcd,efgh} = (\mathcal{R}_x)_{abcd}^{\;mn}(\mathcal{R}_{x'})_{efgh}^{\;pq}N_{mn,pq},$$

$$\mathscr{E}_{\mathrm{EM}} = \sum_{i=X,Y,z}\mathscr{F}_{\tau i,\tau i}, \qquad \mathscr{B}_{\mathrm{EM}} = \frac{1}{4}\sum_i \varepsilon_{ijk}\varepsilon_{ilm}\mathscr{F}_{jk,lm},$$

$$\mathscr{E}_h = \sum_{i,j}\mathscr{R}_{\tau i\tau j,\tau i\tau j}, \qquad \mathscr{B}_h = \frac{1}{4}\sum_{i,j}\varepsilon_{ikl}\varepsilon_{imn}\mathscr{R}_{kl\tau j,mn\tau j}.$$

Each expression is evaluated at $u = v = 0$. The two factors 1/2 in $\mathcal{R}$ are those in .

For checking projector signs, the transverse vector projectors are

$$Q_+ = \frac{1}{2}\begin{pmatrix} 1 & \mathrm{i} \\ -\mathrm{i} & 1 \end{pmatrix}, \qquad Q_- = \overline{Q_+}.$$

The transverse weight-2 unit tensors are

$$v_+ = u_+ \otimes u_+ = \frac{1}{2}\begin{pmatrix} 1 & -\mathrm{i} \\ -\mathrm{i} & -1 \end{pmatrix}, \qquad v_- = \overline{v_+}.$$

The 0 and z vectors have weight zero; symmetric products with $u_{\pm}$ have weight ±1. These facts specify all projectors in , including the symmetric-tensor multiplicities.

Substitution gives the finite contraction table

$c\|c$contraction at$r = 1$	value multiplied by$8\pi^2$
hline $\mathscr{E}_{\mathrm{EM}}$	$(q^4 + 10q^2 - 11)/90$
$\mathscr{B}_{\mathrm{EM}}$	$-(q^4 + 10q^2 - 11)/90$
$\mathscr{E}_h$	$(2q^6 + 21q^4 + 168q^2 - 191)/1260$
$\mathscr{B}_h$	$-(2q^6 + 21q^4 + 168q^2 - 191)/1260$

with $q = \alpha^{-1}$. This is a finite rational-polynomial identity specified by –; it does not assert that a symbolic program was executed.

The same rule with the two reduced spin-2 labels gives –. It also makes clear why discarding odd helicity jets before tensor assembly can change an even final contraction.

Declarations

Authorship. The author name follows the supplied source. The cited Cheung manuscripts are attributed by identifier and supplied version. Citation does not establish external registration of this revision.

Dependency scope. The scalar propagation and supported-transfer input from I-B-G is stated explicitly. The helicity-domain matching, exceptional-channel defect, restricted selection, and finite jet prescription are written out here. Companion comparisons do not import their flat algebras, interacting products, gauge groups, or closure conclusions.

Quantum scope. The Maxwell and graviton brackets are defined free Gaussian contractions in the stated normalization. The gauge-fixed graviton contraction is not asserted to be a positive physical state. No interacting apex algebra or inherited characteristic quantum coefficient is admitted.

Core scope. The curvature-regular criterion is mathematical and idealized. A physical spin-2 core selection is O2. A scalar strong-resolvent core theorem cannot replace a coupled matter–gravity linearization or a one-point limit theorem.

Evidence scope. The supplied numerical reports are preserved as reports. No scripts, raw logs, hashes, or repository paths are asserted to have been independently inspected or rerun. The proposed numerical protocol has not been executed.

Source format and verification. This is one conventional pdfLaTeX source with an internal bibliography and no external figure, code, or data dependency. It is intended for two pdfLaTeX runs. No successful compiler execution, proof-assistant check, independent symbolic-program execution, numerical rerun, hardware compilation, experiment, independent referee verification, or journal acceptance is asserted.

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